



# MANIPUR UNIVERSITY

## Learning Outcomes-based Curriculum Framework (LOCF)

Semester Scheme with Multiple Entry and Exit Options for  
Under Graduate Course



### **Syllabus for Mathematics** **(I, II, III, IV, V & VI Semesters)**

2022-23 onwards

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## Syllabus for Bachelor of Science/Arts in Mathematics

Name of the Degree /Program: Bachelor of Science/ Bachelor of Arts

Discipline Course: Mathematics

Starting year of implementation: 2022-2023

Programme Outcomes (PO): By the end of the program the students will be able to gain the following skills.

PO1	<b>Disciplinary knowledge:</b> Bachelor degree in Mathematics is the culmination of in-depth knowledge of Algebra, Calculus, geometry, Real analysis, Differential equations and several other branches of pure and applied mathematics, This also leads to study of relevant areas such as computer science and other disciplines.
PO2	<b>Communication Skills:</b> Ability to communicate the various mathematical concepts effectively using variety of examples mostly having real life applications and their geometric visualization. The skills and knowledge gained in this programme will lead to the proficiency in analytical reasoning which can be used to express thoughts and views in mathematically or logically correct statements.
PO3	<b>Critical thinking and analytical reasoning:</b> The students undergoing this programme acquire the ability of critical thinking and logical reasoning and will apply in formulating or generalizing specific hypothesis, conclusion. The learner will be able to recognize and distinguish the various aspects of real life problems.
PO4	<b>Problem solving:</b> The Mathematical knowledge gained by the student through this programme develops an ability to solve the problems, identify and define appropriate computing requirements for its solutions. This programme will enhance the overall development.
PO5	<b>Research related skills:</b> After the completion of this programme, the student will develop the capability of inquiring about appropriate questions relating to the Mathematical concepts, arguments. He/she will be able to define problems, formulate hypothesis, proofs, write the results obtained clearly.
PO6	<b>Information/ digital literacy:</b> The completion of this programme will enable the learner to use appropriate softwares to solve the system of algebraic and differential equations.
PO7	<b>Self-directed learning:</b> The student after the completion of the programme will be able to work independently, make an in-depth search of various areas of Mathematics and resources for self learning in order to enhance knowledge in mathematics.
PO8	<b>Moral and ethical awareness / reasoning:</b> The student after the completion of the course will develop an ability to identify unethical behaviour such as fabrication, falsification or misinterpretation of data and adopting objectives, unbiased and truthful actions in all aspects of life in general and Mathematical studies in particular.
PO9	<b>Lifelong learning:</b> This programme provides self directed learning and lifelong learning skills. With these skills, the learner will be able to think independently, improve personal development.

### Assessment

Weightage for the Assessments (in percentage)

Type of Course	Formative Assessment(I.A)	Summative Assessment (S.A)
Theory	25%	75%
Projects	25%	75%
Experimental Learning (Internship etc.)		

## Course Structure (Draft)

### Model A

#### (A) Bachelor's Certificate in Mathematics (Level 5)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (To be selected from GEC listings of other disciplines) (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement Course (SEC) Select any one among A/B	Value Addition Courses (VAC) (To be selected from VAC listings of other disciplines). (Credits)	Semester Credit
I	MMC-101 (6)			AECC-101 (4) English/MIL	MMSE-101 A/B (4)	VAC-101 (2)	24
	MMC-102 (6)					VAC-102 (2)	
II	MMC-203 (6)			AECC-202 (4) Environmental Science	MMSE-202 A/B (4)	VAC-203 (2)	24
	MMC-204(6)					VAC-204 (2)	

**Award of Certificate in Mathematics (after 1<sup>st</sup> Year : minimum 46 (four six) Credits)**

#### (B) Bachelor's Diploma in Mathematics (Level 6)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement Course (SEC)	Value Addition Courses (VAC) (Credits)	Semester Credit
III	MMC-305 (6)		MMGE-301 (6)			VAC-305 (2)	26
	MMC-306 (6)						
	MMC-307 (6)						
IV	MMC-408 (6)		MMGE-402 (6)			VAC-406 (2)	26
	MMC-409 (6)						
	MMC-410 (6)						

**Award of Diploma in Mathematics (after 2nd Year: minimum 96 (nine six) Credits)**

#### (C) Bachelor's Degree in Mathematics (Level 7)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement	Value Addition Courses (VAC) (Credits)	Semester Credit
V	MMC-511 (6)	MME-501 (6)	MMGE-503(6)			VAC-507 (2)	26
	MMC-512 (6)						
VI	MMC-613 (6)	MME-602 (6)	MMGE-604(6)			VAC-608 (2)	26
	MMC-614 (6)						

**Award of BSc degree in Mathematics (after 3rd Year: minimum 140 (one four zero) Credits)**

#### (D) Bachelor's (Hons) Degree (Level 8)

Semester	Discipline specific Core (DSC)	Discipline Specific Elective (DSE)	Generic Elective Course (GEC), (Credit)	Ability Enhancement Compulsory Courses (AECC)	Skill Enhancement Course (SEC)	Value Addition Courses (VAC) (Credits)	Semester Credit
VII	MMC-715 (6)	MME-703 (6)	MMGE-705 (6)				24
	MMC-716 (6)						
VIII	MMC-817 (6)	MME-804 (6) /Research projects	MMGE-806(6)				24
	MMC-818 (6)						

**Award of B A/ B Sc degree with honours in Mathematics on completion of course equal to a minimum of 182 (one eight two) credits.**

Bachelor of Science or Bachelor of Arts with Mathematics as Major & BA/BSc (Hons) Mathematics.

## Course Structure (Draft)

### SEMESTER-WISE DISTRIBUTION OF COURSES

#### A. Discipline Specific Core (DSC) Courses:

All the courses have 6 credits with 4 credits of theory (4 hours per week) and 2 credits of practical (4 hours per week) OR 5 (FIVE) credits of theory and 1(ONE) credit of tutorial

Sl.No.	CC Paper Code	Semester	Course Name
1.	MMC-101	I	Calculus
2.	MMC-102	I	Algebra
3.	MMC-203	II	Real Analysis
4.	MMC-204	II	Differential Equations
5.	MMC-305	III	Theory of Real Functions
6.	MMC-306	III	Group Theory
7.	MMC-307	III	Multivariate Calculus
8.	MMC-408	IV	Partial Differential Equations
9.	MMC-409	IV	Riemann Integration
10.	MMC-410	IV	Numerical Analysis
11.	MMC-511	V	Metric Spaces
12.	MMC-512	V	Mechanics
13.	MMC-613	VI	Complex Analysis
14.	MMC-614	VI	Ring Theory and Linear Algebra
15.	MMC-715	VII	Abstract Algebra
16.	MMC-716	VII	Advanced Real Analysis
17.	MMC-817	VIII	Topology
18.	MMC-818	VIII	Ordinary Differential Equations

#### B. Discipline Specific Electives (DSE):

All the courses have 6 credits with 4 credits of theory and 2 credits of practical or 5 credits of theory and 1 credit of tutorials.

Sl. No.	DSE Paper Code	Semester	DSE Name
1.	MME-501	V	Integral Transform/Mathematical Modelling/Advanced Group Theory
2.	MME-602	VI	Special Theory of Relativity & Tensor/Linear Programming and its applications/Probability Theory and Statistics
3.	MME-703	VII	Advanced Complex Analysis/Graph Theory/Fixed point theory
4.	MME-804	VIII	Advanced Partial Differential Equations/Functional Analysis/Cryptology/Research Projects

**C. Skill Enhancement Courses (SEC):**

All courses have 4 credits with 2 credits of theory and 2 credits of Practical/Tutorials/Projects and Field Work to be decided by the College.

Sl No.	DSE Paper Code	Semester	SEC Name
1.	MMSE-101 A	I	LaTeX
2.	MMSE-101 B	I	Computational Mathematics Laboratory
3.	MMSE-202 A	II	Python Programming
4.	MMSE-202 B	II	Computer Algebra Systems and Related Software

**D. Ability Enhancement Compulsory Courses:**

All the courses have 4 credits including Theory/Practicals/Projects.

Sl No.	AECC Paper Code	Semester	AECC Name
1.	AECC-101	I	English/MIL
2.	AECC-202	II	Environmental Science

**E. Value Addition Courses:**

Sl.No	VAC Paper Code	Semester	VAC Name
1.	VAC-101	I	Yoga
2.	VAC-102	I	Sports
3.	VAC-203	II	Culture
4.	VAC-204	II	Health Care
5.	VAC-305	III	NCC
6.	VAC-406	IV	Ethics
7.	VAC-507	V	NSS
8.	VAC-608	VI	History of Science

**F. Generic Elective Courses:**

All the courses have 6 credits with 4 credits of theory and 2 credits of practicals. These courses are meant for students of other departments/disciplines or 5 credits of theory and 1 credit of tutorials.

Sl.No.	GECPaperCode	Semester	GEC Name
1.	MMGE-301	III	Quantitative Aptitude
2.	MMGE-402	IV	Basic Tools of Mathematics
3.	MMGE-503	V	Recreational Mathematics
4.	MMGE-604	VI	Discrete Mathematics
5.	MMGE-705	VII	Analytical Geometry and Theory of Equations
6.	MMGE-806	VIII	Numerical methods with practical

## Contents of Courses for B.A./B.Sc. degree with honours in Mathematics Model B (Revised)

Semester	Course Code		Credit	Paper Title	Marks		Remark	
					S.A	I.A.		
I	MMC-101	Theory	6	Calculus	75	25	Approved with Syllabus	
	MMC-102	Theory	6	Algebra	75	25		
	MMSE-101(A/B)	Theory& practical	4	Latex /Computational Mathematics Laboratory	37.5 37.5	12.5 12.5		
II	MMC-203	Theory	6	Real Analysis	75	25		
	MMC-204	Theory	6	Differential Equations	75	25		
	MMSE-202 (A/B)	Theory& Practical	4	Python Programming/Computer Algebra Systems and related Software	37.5 37.5	12.5 12.5		
<b>Exit Option with Certificate</b>								
III	MMC-305	Theory	6	Theory of Real Functions	75	25		Approved with Syllabus
	MMC-306	Theory	6	Group Theory	75	25		
	MMC-307	Theory	6	Multivariate Calculus	75	25		
	MMGE-301	Theory	6	Quantitative Aptitude	75	25		
IV	MMC-408	Theory	6	Partial Differential Equations	75	25		
	MMC-409	Theory	6	Riemann Integration	75	25		
	MMC-410	Theory	6	Numerical Analysis	75	25		
	MMGE-402	Theory	6	Basic Tools of Mathematics	75	25		
<b>Exit Option with Diploma</b>								
V	MMC-511	Theory	6	Metric Spaces	75	25	To be approved in subsequent BOS	
	MMC-512	Theory	6	Mechanics	75	25		
	MME-501	Theory	6	Integral Transform/Mathematical Modelling/ Advanced Group Theory	75	25		
	MMGE-503	Theory	6	Recreational Mathematics	75	25		
VI	MMC-613	Theory	6	Complex Analysis	75	25		
	MMC-614	Theory	6	Ring Theory and Linear Algebra	75	25		
	MME-602	Theory	6	Special Theory of Relativity & Tensor/Linear Programming and its applications/Probability Theory and Statistics	75	25		
	MMGE-604	Theory	6	Discrete Mathematics	75	25		
<b>Exit Option with Bachelor of Arts, B.A./Bachelor of Science, B.Sc.</b>								
VII	MMC-715	Theory	6	Abstract Algebra	75	25	To be approved in subsequent BOS	
	MMC-716	Theory	6	Advanced Real Analysis	75	25		
	MME-703	Theory	6	Advanced Complex Analysis/Graph Theory/ Fixed Point Theory	75	25		
	MMGE-705	Theory	6	Analytic Geometry and Theory of Equations	75	25		
VIII	MMC-817	Theory	6	Topology	75	25		
	MMC-818	Theory	6	Ordinary Differential Equations	75	25		
	MME-804	Theory	6	Advanced Partial Differential Equations/Functional Analysis/ Cryptology/RESEARCH PROJECTS	75	25		
	MMGE-806	Theory	6	Numerical Methods with practical	75	25		



## Discipline Specific Core Courses Semester I

### MMC-101: Calculus

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week)

**Duration:** 14 Weeks (70 Hrs. ) **Examination:** 3 Hrs.

**Course Objectives:** The primary objective of this course is to introduce the basic tools of calculus and geometric properties of different conic sections which are helpful in understanding their applications in planetary motion, design of telescope and to the real-world problems. Also, to carry out the hand on sessions in computer lab to have a deep conceptual understanding of the above tools to widen the horizon of students' self-experience.

**Course Learning Outcomes:** After completion of the course, a student will be able to:

- i) sketch curves in a plane in the different coordinate systems of reference.
- ii) understand the Calculus of vector valued functions.
- iii) apply calculus to develop basic principles of planetary motions.

#### Unit 1: Derivatives for Curve sketching

(35 marks, 5 weeks)

First and second derivative tests for Extreme Values of Functions, Concavity and Curve Sketching, Limits to infinity and infinite limits, Indeterminate Forms and L'Hôpital's Rule, Asymptotes, Higher order derivatives, Leibniz rule.

#### Unit 2: Curve tracing in polar Co-ordinates

(30 marks, 4 weeks)

Parametric representation of curves, Polar Coordinates, Tracing of curves in Polar Coordinates, Graphing Polar Coordinate Equations, Areas and Lengths in Polar Coordinates, Classification of conics in Polar Coordinates.

#### Unit 3: Vector Calculus and its applications

(35 marks, 5 weeks)

Vector valued functions and their graphs, Limits and continuity of vector functions, Differentiation and integration of vector functions, Projectile motion, Unit tangent, Normal and binormal vectors, Curvature, Kepler's Second Law (Equal Area Law).

#### References:

1. Thomas, Jr. George B., Weir, Maurice D., & Hass, Joel (2014). Thomas' Calculus (13<sup>th</sup> ed.) Pearson Education, Delhi. Indian Reprint 2017.
2. B. C. Das, B. N. Mukherjee. Differential Calculus (55<sup>th</sup> Edition), U.N. Dhur & Sons Private Ltd., Kolkata (2015).

## Teaching plan ( MMC 101 Calculus):

- Week 1:** First and second derivative tests for Extreme Values of Functions; [1] Chapter 4 (Section 4.3).  
**Week 2:** Concavity and Curve Sketching; [1] Chapter 4 (Section 4.4).  
**Week 3:** Limits to infinity and infinite limits; [1] Chapter 2 (Section 2.6).  
**Week 4:** Indeterminate forms and L'Hospital's Rule; [1] Chapter 4 (Section 4.5), Asymptotes; [1] Chapter 2 (Section 2.6).  
**Week 5:** Higher order derivatives; [1] Chapter 3 (Section 3.7), Leibniz rule; [1] Chapter 3 (Section 3.11).  
**Week 6:** Parametric representation of curves; [1] Chapter 11 (Section 11.1 and 11.2),  
**Week 7:** Polar Coordinates, Tracing of curves in Polar Coordinates; [1] Chapter 11 (Section 11.3).  
**Week 8:** Graphing Polar Coordinates Equations; [1] Chapter 11 (Section 11.4), Areas and Lengths in Polar Coordinates; [1] Chapter 11 (Section 11.5).  
**Week 9:** Classification of Conics in Polar Coordinates; [1] Chapter 11 (Section 11.6 and 11.7).  
**Week 10:** Vector valued functions and their graphs, Limits and Continuity of vector functions; [1] Chapter 13 (Section 13.1).  
**Week 11:** Differentiation and integration of vector functions; [1] Chapter 13 (Section 13.1).  
**Week 12:** Projectile motion; [1] Chapter 13 (Section 13.2), Unit tangent; [1] Chapter 13 (Section 13.3).  
**Week 13:** Normal and binormal vectors; [1] Chapter 13 (Section 13.3).  
**Week 14:** Curvature; [1] Chapter 13 (Section 13.4 and 13.5), Kepler's Second Law (Equal Area Law); [1] Chapter 13 (Section 13.6).

## MMC-102: Algebra

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The primary objective of this course is to introduce the basic tools of theory of equations, complex numbers, number theory and matrices to understand their linkage to the real-world problems.

**Course Learning Outcomes:** After completion of the course, a student will be able to

- Employ De Moivre's theorem in a number of applications to solve numerical problems;
- Apply Euclid's algorithm and backwards substitution to find greatest common divisor;
- Recognize consistent and inconsistent systems of linear equations by using rank.

### Unit 1: Theory of Equations

(35 marks, 5 weeks)

Polynomial functions, Division algorithm, Synthetic division, Remainder Theorem, Factor Theorem, Polynomial equations, Relation between roots and Co-efficients of a polynomial equation, Symmetric function of the roots of an equation, sum of powers of the roots, Solution of cubic and biquadratic equations, De Moivre's Theorem for integer and fractional indices.

**Unit2: Relation, functions and Basic Number Theory**

**(35 marks, 5 weeks)**

Binary relations, Partial order relation, Equivalence relations, Functions, Inverses and composition, One to one correspondence and Cardinality of a set, Division Algorithm, Divisibility and the Euclidean Algorithm, Prime Numbers, Congruences and applications, Principles of Mathematical induction.

**Unit 3: Matrices**

**(30 marks, 4 weeks)**

Rank of a matrix, Rank and elementary operations, Row reduction and echelon forms, System of linear equations, Solution of the matrix equation  $AX=B$ , Solution sets of linear systems, linear independence, Eigenvectors and Eigen values, The Characteristic equation and Cayley- Hamilton Theorem.

**References:**

1. Goodaire, Edgar G &Parmentor, Michael M (2005); Discrete Mathematics with Graph Theory (3rd Ed.) Pearson Education Pvt. Ltd., Indian Reprint 2015
2. MK Singal, Asha Rani Singal, (2020); Algebra (31<sup>st</sup> Ed) R Chand &Co, New Delhi.
3. Chandrika Prasad, (1963). Text Book on Algebra and Theory of Equations PothishalaPvt. Ltd.

**Additional Readings:**

1. Kolman, Bernard, & Hill, David R. (2001). *Introductory Linear Algebra with Applications* (7th ed.). Pearson Education, Delhi. First Indian Reprint 2003.
2. Lay, David C., Lay, Steven R., & McDonald, Judi J. (2016). *Linear Algebra and its Applications* (5th ed.). Pearson Education.
3. Andrilli, Stephen, &Hecker, David (2016). *Elementary Linear Algebra* (5th ed.). Academic Press, Elsevier India Private Limited.
4. Burton, David M. (2007). *Elementary Number Theory* (7th ed.). Tata Mc-Graw Hill Edition, Indian Reprint.

**Teaching plan ( MMC 102 Algebra):**

**Week 1:** Polynomial functions, Division Algorithm, Synthetic division; [2] Chapter 3 (Section 3.2, 3.3 &3.4).

**Week 2:**Remainder Theorem, Factor Theorem;[1] Chapter 4 (Section 4.1),

**Week 3:**Polynomial equations, Relations between roots and Co-efficients of a polynomial equation; [2] Chapter 3 (Section 3.6 &3.7).

**Week 4:**Symmetric functions of the roots of an equation, Sum of the powers of the roots; [2] Chapter 3 (Section 3.10 &3.10), Solutions of cubic and biquadratic equations; [3]Chapter 13(Section 13.2, 13.3, 13.6, 13.7)

**Week 5:**DeMoivre's Theorem for integer and fractional indices; [2] Chapter 4 (Section 4.1).

**Week 6:**Binary relations, Partial order relation, Equivalence relations; [1] Chapter 2 (Section 2.3 &2.4).

**Week 7:**Functions,Domain, Range, One-One, Onto, Inverses and composition, One to One correspondence and Cardinality of a set; [1] Chapter 3 (Section 3.1, 3.2 &3.3).

**Week 8:**Division Algorithm, Divisibility and The Euclidean Algorithm;[1] Chapter 4 (Section 4.2).

**Week 9:**Prime Numbers, Congruences and applications; [1] Chapter 4 (Section 4.3 & 4.4).

**Week 10:**Principle of Mathematical Induction; [1] Chapter 5 (Section 5.1).

**Week 11:**Rank of a matrix, Rank and elementary operations; [2] Chapter 6 (Section 6.2 & 6.3).

**Week 12:**System of linear equations, Solution of the matrix equation  $AX=B$ ; [2] Chapter 7 (Section 7.2 &7.3)

**Week 13:**Solution sets of linear systems, linear independence; [2] Chapter 6 (Section 6.4), Eigenvectors and Eigen values; [2] Chapter 8 (Section 8.2).

**Week 14:**The Characteristic equation and Cayley-Hamilton Theorem; [2] Chapter 8 (Section 8.4).

## Semester II

### MMC-203: Real Analysis

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The course will develop a deep and rigorous understanding of real line and of defining terms to prove the results about convergence and divergence of sequences and series of real numbers. These concepts have a wide range of applications in real life scenarios.

**Course Learning Outcomes:** This course will enable the students to:

- i) Understand many properties of the real line  $\mathbb{R}$  and learn to define sequence in terms of functions from a subset of  $\mathbb{R}$ .
- ii) Recognize bounded, convergent, divergent, Cauchy and monotonic sequences and to calculate their limit superior, limit inferior, and the limit of a bounded sequence.
- iii) Apply the ratio, root, alternating series and limit comparison tests for convergence and absolute convergence of an infinite series of real numbers.

#### Unit 1: Real Number System and its properties

(30 Marks, 4 Weeks)

Algebraic and order properties of  $\mathbb{R}$ , Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of  $\mathbb{R}$ , the completeness property of  $\mathbb{R}$ , Archimedean property, Density of rational numbers in  $\mathbb{R}$ ; Definition and types of intervals, Nested intervals property; Neighbourhood of a point in  $\mathbb{R}$ , Open and closed sets in  $\mathbb{R}$ .

#### Unit 2: Sequences in $\mathbb{R}$

(35 Marks, 5 Weeks)

Convergent sequence, Limit of a sequence, Bounded sequence, Limit theorems, Monotone sequences, Monotone convergence theorem, Subsequences, Bolzano-Weierstrass theorem for sequences, Limit superior and limit inferior for bounded sequence, Cauchy sequence, Cauchy's convergence criterion.

#### Unit 3: Infinite Series

(35 Marks, 5 Weeks)

Convergence and divergence of infinite series of real numbers, Necessary condition for convergence, Cauchy criterion for convergence; Tests for convergence of positive term series: Integral test, Basic comparison test, Limit comparison test, D'Alembert's ratio test, Cauchy's  $n^{\text{th}}$  root test; Alternating series, Leibniz test, Absolute and conditional convergence.

#### References:

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition. New Delhi.
2. Ross, Kenneth A. (2013). *Elementary Analysis: The theory of calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.
3. Denlinger, Charles G. (2011). *Elements of Real Analysis*. Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.

**Additional Readings:**

1. Bilodeau, Gerald G., Thie, Paul R., & Keough, G. E. (2010). *An Introduction to Analysis* (2nd ed.). Jones and Bartlett India Pvt. Ltd. Student Edition. Reprinted 2015.
2. Thomson, Brian S., Bruckner, Andrew. M., & Bruckner, Judith B. (2001). *Elementary Real Analysis*. Prentice Hall.

**Teaching Plan ( MMC-203: Real Analysis):**

**Weeks 1 and 2:** Algebraic and order properties of  $\mathbb{R}$ . Absolute value of a real number; Bounded above and bounded below sets, Supremum and infimum of a nonempty subset of  $\mathbb{R}$ .

[1] Chapter 2 [Sections 2.1, 2.2 (2.2.1 to 2.2.6), and 2.3 (2.3.1 to 2.3.5)]

**Weeks 3 and 4:** The completeness property of  $\mathbb{R}$ , Archimedean property, Density of rational numbers in  $\mathbb{R}$ ; Definition and types of intervals, Nested intervals property; Neighborhood of a point in  $\mathbb{R}$ , Open and closed sets in  $\mathbb{R}$ .

[1] Chapter 2 [Sections 2.3 (2.3.6), 2.4 (2.4.3 to 2.4.9), and 2.5 up to Theorem 2.5.3]

[1] Chapter 11 [Section 11.1 (11.1.1 to 11.1.3)]

**Weeks 5 and 6:** Convergent sequence, Sequences and their limits, Bounded sequence, Limit theorems.

[1] Chapter 3 (Sections 3.1 and 3.2)

**Week 7:** Monotone sequences, Monotone convergence theorem and applications.

[1] Chapter 3 (Section 3.3)

**Week 8:** Subsequences and statement of the Bolzano-Weierstrass theorem, Limit superior and limit inferior for bounded sequence of real numbers with illustrations only.

[1] Chapter 3 [Section 3.4 (3.4.1 to 3.4.12), except 3.4.4, 3.4.7, 3.4.9 and 3.4.11]

**Week 9:** Cauchy sequences of real numbers and Cauchy's convergence criterion.

[1] Chapter 3 [Section 3.5 (3.5.1 to 3.5.6)]

**Week 10:** Convergence and divergence of infinite series, Sequence of partial sums of infinite series, Necessary condition for convergence, Cauchy criterion for convergence of series.

[3] Chapter 8 (Section 8.1)

**Weeks 11 and 12:** Tests for convergence of positive term series: Integral test statement and convergence of  $p$ -series, Basic comparison test, Limit comparison test with applications, D'Alembert's ratio test and Cauchy's  $n$ th root test.

[3] Chapter 8 (Section 8.2 up to 8.2.19)

**Weeks 13 and 14:** Alternating series, Leibniz test, Absolute and conditional convergence.

[3] Chapter 8 [Section 8.3 (8.3.1 to 8.3.7)]

## **MMC-204: Differential Equations**

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (56 Hrs. Theory) **Examination:** 3 Hrs.

**Course Objectives:** The main objectives of this course are to introduce the students to the exciting world of Differential Equations, Mathematical Modeling and their applications.

**Course Learning Outcomes:** The course will enable the students to:

- i) Formulate Differential Equations for various Mathematical models.
- ii) Solve first order non-linear differential equation and linear differential equations of higher order using various techniques.
- iii) Apply these techniques to solve and analyze various mathematical models.

**Unit 1: Differential Equations and Mathematical Modeling**

**(35 marks, 5 weeks)**

Differential equations and mathematical models, Order and degree of a differential equations, Integrals as general and particular solutions, Exact differential equations and integrating factors of first order differential equations, Separable Equations, Homogeneous Equations, Reduction to homogeneous equations, Linear equations and Bernoulli Equation, Clairaut's Equation, Existence and Uniqueness of solution of initial and boundary value problems of first order ODE, singular solution of first order ODE.

**Unit 2: Second and higher order differential Equations**

**(35 marks, 5 weeks)**

General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation, Wronskian, its properties and applications, Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation, Method of undetermined coefficients, Method of variation of parameters, Applications of second order differential equations to mechanical vibration.

**Unit 3: Analysis of Mathematical Models**

**(30 marks, 4 weeks)**

Application of first order differential equations to acceleration-velocity model, Growth and Decay model. Introduction to compartmental models, Lake pollution model (with case study of Lake Burley Griffin), Drug Assimilation models, population models (with limited growth, exponential growth) Epidemic models.

**References:**

1. Barnes, Belinda & Fulford, Glenn R. (2015). *Mathematical Modelling with Case Studies, Using Maple and MATLAB* (3rd ed.). CRC Press, Taylor & Francis Group.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). *Differential Equation and Boundary Value Problems: Computing and Modeling* (5th ed.). Pearson Education.
3. Ross, Shepley L. (2004). *Differential Equations* (3rd ed.). John Wiley & Sons. India.

**Teaching plan (MMC-204 : Differential Equations)**

**Week 1:** Differential equations and mathematical models; [2] Chapter 1 (Section 1.1), Order and degree of a differential equations; [3] Chapter 1 (Section 1.1)

**Week 2:** Integrals as general and particular solutions; [2] Chapter 1 (Section 1.2), Exact differential equations and integrating factors of first order differential equations; [3] Chapter 2 (Section 2.1)

**Week 3:** Separable equations, Homogeneous equations, Reduction to homogeneous equations; [3] Chapter 2 (Section 2.2).

**Week 4:** Linear equations and Bernoulli equation Clairaut's equation ; [3] Chapter 2 (Section 2.3)

**Week 5:** Existence and Uniqueness of solution of initial and boundary value problems of first order ODE; singular solution of the first order ODE [3] Chapter 1 (Section 1.3),

**Week 6 & 7:** General solution of homogeneous equation of second order, Principle of superposition for a homogeneous equation, Wronskian, its properties and applications; [2] Chapter 3 (Section 3.1).

**Week 8:** Linear homogeneous and non-homogeneous equations of higher order with constant coefficients, Euler's equation; [2] Chapter 3 (Section 3.3).

**Week 9:** Method of undetermined coefficients, Method of variation of parameters; [2] Chapter 3 (Section 3.5).

**Week 10:** Applications of second order differential equations to mechanical vibration; [2] Chapter 3 (Section 3.6).

**Week 11& 12:** Application of first order differential equations to acceleration-velocity model; [2] Chapter 2 (Section 2.3), Growth and Decay model; [1] Chapter 2 (Section 2.2).

**Week 13:** Introduction to compartmental models; [1] Chapter 2 (Section 2.1), Lake pollution model (with case study of Lake Burley Griffin); [1] Chapter 2 (Section 2.5 & 2.6), Drug Assimilation models; [1] Chapter 2 (Section 2.7).

**Week 14:** Population models (with limited growth, exponential growth) Epidemic models; [2] Chapter 2 (Section 2.1) or [1] Chapter 3 (Section 3.1)

## **Semester III**

### **MMC-305: Theory of Real Functions**

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** It is a basic course on the study of real valued functions that would develop analytical ability to have a more matured perspective of the key concepts of calculus, namely, limits, continuity, differentiability and their applications.

**Course Learning Outcomes:** This course will enable the students to learn:

- i) A rigorous approach of the concept of limit of a function.
- ii) About continuity and uniform continuity of functions defined on intervals.
- iii) The geometrical properties of continuous functions on closed and bounded intervals.
- iv) The applications of mean value theorem and Taylor's theorem.

#### **Unit 1: Limits of Functions**

**(20 Marks, 3 Weeks)**

Limits of functions ( $\epsilon$ - $\delta$  approach), Sequential criterion for limits, Divergence criteria, Limit theorems, One-sided limits, Infinite limits and limits at infinity.

#### **Unit 2: Continuous Functions and their Properties**

**(35 Marks, 5 Weeks)**

Continuous functions, Sequential criterion for continuity and discontinuity, Algebra of continuous functions, Properties of continuous functions on closed and bounded intervals; Uniform continuity, Non-uniform continuity criteria, Uniform continuity theorem.

#### **Unit 3: Derivability and its Applications**

**(45 Marks, 6 Weeks)**

Differentiability of a function, Algebra of differentiable functions, Carathéodory's theorem and chain rule; Relative extrema, Interior extremum theorem, Rolle's theorem, Mean-value theorem and its applications, Intermediate value property of derivatives - Darboux's theorem, Taylor polynomial, Taylor's theorem with Lagrange form of remainder, Application of Taylor's theorem in error estimation; Relative extrema, and to establish a criterion for convexity; Taylor's series expansions of  $e^x$ ,  $\sin x$  and  $\cos x$

#### **Reference:**

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4<sup>th</sup> ed.). Wiley India Edition. New Delhi.

**Additional Readings:**

1. Ghorpade, Sudhir R. & Limaye, B. V. (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
2. Mattuck, Arthur. (1999). *Introduction to Analysis*, Prentice Hall.
3. Ross, Kenneth A. (2013). *Elementary Analysis: The theory of calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer. Indian Reprint.

**MMC-306: Group Theory**

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The objective of the course is to introduce the fundamental theory of groups and their homomorphisms. Symmetric groups and group of symmetries are also studied in detail. Fermat's Little theorem as a consequence of the Lagrange's theorem on finite groups.

**Course Learning Outcomes:** After completion of the course, a student will be able to

- i) understand the basic concepts of groups and links with symmetric figures;
- ii) learn concepts of normal subgroups, cosets and quotient groups;
- iii) learn the concepts of group homomorphisms and isomorphisms.

**Unit 1: Groups and elementary properties**

**(35 Marks, 5 Weeks)**

Symmetries of a Square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups, cycle notation of permutations, properties of permutations, Elementary properties of groups, Permutations, Even and odd permutations.

**Unit 2: Subgroups**

**(35 Marks, 5 Weeks)**

Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a group, Cosets of a Group, Lagrange's theorem and consequences including Fermat's Little theorem, cyclic groups, Classification of subgroups of cyclic groups, Normal subgroups, Quotient Groups, alternating groups.

**Unit 3: Group Homomorphisms**

**(30 Marks, 4 Weeks)**

Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Properties of isomorphisms, First, Second and Third isomorphism theorems for groups, Cayley's theorem

**Reference**

1. Gallian, Joseph. A. (2013). *Contemporary Abstract Algebra* (8th ed.). Cengage Learning India Private Limited, Delhi. Fourth impression, 2015.
2. I.N. Herstein, (2006). *Topics in Algebra* (2<sup>nd</sup> Edn). Wiley India Pvt. Ltd.



**Additional Reading:**

1. V.K.Khanna, SK Bhambri (2017). A course in Abstract Algebra (5<sup>th</sup>Edn).Vikas Pub. House Pvt Ltd.
2. Rotman, Joseph J. (1995). *An Introduction to The Theory of Groups* (4th ed.). Springer Verlag, NY.

**Teaching Plan (MMC-306: Group Theory):**

**Week 1:** Symmetries of a square, Dihedral groups, Definition and examples of groups including permutation groups and quaternion groups (illustration through matrices). [1] Chapter 1.

**Week 2:** Definition and examples of groups, Elementary properties of groups. [1] Chapter 2.

**Week 3:** Subgroups and examples of subgroups, Centralizer, Normalizer, Center of a Group, Product of two subgroups. [1] Chapter 3.

**Weeks 4 and 5:** Properties of cyclic groups. Classification of subgroups of cyclic groups. [1] Chapter 4

**Weeks 6 and 7:** Cycle notation for permutations, Properties of permutations, Even and odd permutations, [1] Chapter 5 (up to Page 110).

**Weeks 8 and 9:** Properties of cosets, Lagrange's theorem and consequences including Fermat's Little theorem. [1] Chapter 7 (up to Example 6, Page 150).

**Week 10:** Normal subgroups, Factor groups, Cauchy's theorem for finite abelian groups. [1] Chapters 9 (Theorem 9.1, 9.2, 9.3 and 9.5, and Examples 1 to 12), Alternating group, [1] Chapter 5 (up to Page 110).

**Weeks 11 and 12:** Group homomorphisms, Properties of homomorphisms, Group isomorphisms, Cayley's theorem. [1] Chapter 10 (Theorems 10.1 and 10.2, Examples 1 to 11). [1] Chapter 6 (Theorem 6.1, and Examples 1 to 8).

**Weeks 13 and 14:** Properties of isomorphisms, First, Second and Third isomorphism theorems. [1] Chapter 6 (Theorems 6.2 and 6.3), Chapter 10 (Theorems 10.3, 10.4, Examples 12 to 14, and Exercises 41 and 42 for second and third isomorphism theorems for groups).

## MMC-307: Multivariate Calculus

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (70 Hrs. Theory) **Examination:** 3 Hrs.

**Course Objectives:** To understand the extension of the studies of single variable differential and integral calculus to functions of two or more independent variables. Also, the emphasis will be on the use of Computer Algebra Systems by which these concepts may be analyzed and visualized to have a better understanding.

**Course Learning Outcomes:** This course will enable the students to learn:

- i) The conceptual variations when advancing in calculus from one variable to multivariable discussions.
- ii) Inter-relationship amongst the line integral, double and triple integral formulations.
- iii) Applications of multi variable calculus tools in physics, economics, optimization, and understanding the architecture of curves and surfaces in plane and space etc.

**Unit 1: Calculus of Functions of Several Variables and Properties of Vector Field- (40 Marks, 6 weeks)**

Functions of several variables, Level curves and surfaces, Limits and continuity, Partial differentiation, Higher order partial derivative, Tangent planes, Total differential and differentiability, Chain rule, Directional derivatives, The gradient, Maximal and normal property of the gradient, Tangent planes and normal lines, Extrema of functions of two variables, Method of Lagrange multipliers, Constrained optimization problems; Definition of vector field, Divergence and curl.

**Unit 2: Double and Triple Integrals – (30 Marks, 4 Weeks)**

Double integration over rectangular and nonrectangular regions, Double integrals in polar co-ordinates, Triple integral over a parallelepiped and solid regions, Volume by triple integrals, triple integration in cylindrical and spherical coordinates, Jacobians (Without Proof), Change of variables in double and triple integrals.

**Unit 3: Green's, Stokes' and Gauss Divergence Theorem – (30 Marks, 4 Weeks)**

Line integrals, Applications of line integrals: Mass and Work, Fundamental theorem for line integrals, Conservative vector fields, Green's theorem, Area as a line integral; Surface integrals, Stokes' theorem, The Gauss divergence theorem.

**References:**

1. Strauss, Monty J., Bradley, Gerald L., & Smith, Karl J. (2007). *Calculus* (3rd ed.). Dorling Kindersley (India) Pvt. Ltd. (Pearson Education). Delhi. Indian Reprint 2011.
2. Marsden, J. E., Tromba, A., & Weinstein, A. (2004). *Basic Multivariable Calculus*. Springer (SIE). First Indian Reprint.

**Teaching Plan (MMC-307: Multivariate Calculus):**

**Week 1:** Definition of functions of several variables, Graphs of functions of two variables-Level curves and surfaces, limits and continuity of functions of two variables.

[1] Chapter 11(Sections 11.1 and 11.2)

**Week 2:** Partial differentiation, and partial derivative as slope and rate, Higher order partial derivatives, Tangent Planes, incremental approximation, Total differential.

[1] Chapter 11 (Sections 11.3 and 11.4)

**Week 3:** Differentiability, Chain rule for one parameter, Two and three independent parameters.

[1] Chapter 11 (Section 11.4 and 11.5)

**Week 4:** Directional derivatives, The gradient, Maximal and normal property of the gradient, Tangent planes and normal lines.

[1] Chapter 11 (Section 11.6)

**Week 5:** First and second partial derivative tests for relative extrema of functions of two variables, and absolute extrema of continuous functions.

[1] Chapter 11(section 11.7 (upto page 605))

**Week 6:** Lagrange multipliers method for optimization problems with one constraint, Definition of vector field, Divergence and curl.

[1] Chapter 11 [Section 11.8 (pages 610-614) Chapter 13 (section 13.1)

**Week 7:** Double integration over rectangular and nonrectangular regions.

[1] Chapter 12(Sections 12.3 and 12.4)

**Week 8:** Double integrals in polar coordinates, and triple integral over a parallelepiped.

[1] Chapter 12 (Sections 12.3 and 12.4)

**Week 9:** Triple integral over solid regions, Volume by triple integral, and triple integration in cylindrical coordinates.

[1] chapter 12 (Sections 12.4 and 12.5)

**Week 10:** Triple integration in spherical coordinates, Jacobian (Without Proof), Change of variables in double and triple integrals.

[1] Chapter 12(Sections 12.7 and 12.8 upto page 849)

**Week 11:** Line integrals and its properties, applications of line integrals: mass and work.

[1] Chapter 13 (Section 13.2)

**Week 12:** Fundamental theorem for line integrals, Conservative vector fields and path independence.

[1] Chapter 13 (Section 13.3)

**Week 13:** Green's theorem for simply connected region, area as a line integral, Definition of surface integrals.

[1] Chapter 13 (Sections 13.4 and 13.7)

**Week 14:** Stokes' theorem and the divergence theorem.

[1] Chapter 13 (Sections 13.6 and 13.7)

## Semester IV

### MMC-408: Partial Differential Equations

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (56 Hrs. Theory) **Examination:** 3 Hrs.

**Course Objectives:** The main objectives of this course are to teach students to form and solve partial differential equations and use them in solving some physical problems.

**Course Learning Outcomes:** The course will enable the students to

- i. Formulate, classify and transform partial differential equations into canonical form
- ii. Solve linear and non-linear partial differential equations using various methods: and apply these methods in solving some physical problems.

#### Unit 1. First order PDE and Methods of Characteristics

(30 Marks, 4 Weeks)

Definitions & Basic concepts, Formation of PDE, classification and geometrical interpretation of first order partial differential equations (PDE), Method of characteristics and general solution of first order PDE, Lagrange and Charpit method, Cauchy's problems for first order PDE, Canonical form of first order PDE, Method of separation of variables for first order PDE

#### Unit 2. Classification of second order Linear PDE an Wave equations

(35 Marks, 5 Weeks)

Classification of second order PDE, Reduction to canonical forms, Equations with constant coefficients, General solutions, Cauchy's Problem for second order PDE, Mathematical Modeling of vibrating string, vibrating membrane, Homogeneous wave equation, Initial boundary value problems, Non-homogenous boundary conditions, Finite string with fixed ends, Non- homogeneous wave equation.

**Unit 3. Methods of separation of Variables**

**(35 Marks, 5 Weeks)**

Methods of separation of Variables for second order PDE, vibrating string problems, Existence and uniqueness of solution of vibrating string problems, Heat conduction problem, Existence and uniqueness of solution of Heat conduction problems, General solution of higher order PDE with constant coefficient, Non-homogeneous Problems.

**References:**

1. Myint-U, Tyn and Debnath, Lokenath. (2007). *Linear Partial Differential Equation for Scientists and Engineers* (4<sup>th</sup>ed). Springer, Third Indian Reprint.

**Additional Readings:**

1. Sneddon, I. N. (2006). *Elements of Partial Differential Equations*, Dover Publications. Indian Reprint.
2. Stavroulakis, Ioannis P & Tersian, Stepan A. (2004). *Partial Differential Equations: An Introduction with Mathematica and MAPLE* (2nd ed.). World Scientific.

**Teaching Plan (Theory of MMC-408: Partial Differential Equations):**

**Week 1:** Introduction, Classification, Construction of first order partial differential equations (PDE).

[1] Chapter 2 (Sections 2.1 to 2.3)

**Week 2:** Method of characteristics and general solution of first order PDE.

[1] Chapter 2 (Sections 2.4 and 2.5)

**Week 3:** Canonical form of first order PDE, Method of separation of variables for first order PDE.

[1] Chapter 2 (Sections 2.6 and 2.7)

**Week 4:** The vibrating string, Vibrating membrane, Gravitational potential, Conservation laws.

[1] Chapter 3 (Sections 3.1 to 3.3, 3.5, and 3.6)

**Weeks 5 and 6:** Reduction to canonical forms, Equations with constant coefficients, General solution.

[1] Chapter 4 (Sections 4.1 to 4.5)

**Weeks 7 and 8:** The Cauchy problem for second order PDE, Homogeneous wave equation.

[1] Chapter 5 (Sections 5.1, 5.3, and 5.4)

**Weeks 9 and 10:** Initial boundary value problem, Non-homogeneous boundary conditions, Finite string with fixed ends, Non-homogeneous wave equation, Goursat problem.

[1] Chapter 5 (Sections 5.5 to 5.7, and 5.9)

**Weeks 11 and 12:** Method of separation of variables for second order PDE, Vibrating string problem.

[1] Chapter 7 (Sections 7.1 to 7.3)

**Weeks 13 and 14:** Existence (omit proof) and uniqueness of vibrating string problem. Heat conduction problem. Existence (omit proof) and uniqueness of the solution of heat conduction problem. Non-homogeneous problem.

[1] Chapter 7 (Sections 7.4 to 7.6, and 7.8)

## MMC-409: Riemann Integration

**Total Marks:** 100 (Theory: 70 and Internal Assessment: 30)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** To understand the integration of bounded functions on a closed and bounded interval and its extension to the cases where either the interval of integration is infinite, or the integrand has infinite limits at a finite number of points on the interval of integration. The sequence and series of real valued functions, and an important class of series of functions (i.e., power series).

**Course Learning Outcomes:** The course will enable the students to learn about:

- i) Some of the families and properties of Riemann integrable functions, and the applications of the fundamental theorems of integration.
- ii) Beta and Gamma functions and their properties.
- iii) The valid situations for the inter-changeability of differentiability and integrability within infinite sum, and approximation of transcendental functions in terms of power series.

### Unit 1: Riemann Integration

(35 Marks, 5 Weeks)

Definition of Riemann integration, (Algebraic and order properties of Riemann Integrals) Boundedness theorem, Riemann integrability, Cauchy's criterion, Squeeze Theorem, Riemann integrability of step, continuous, and monotone functions, Additivity theorem, Fundamental theorems (First and Second forms), substitution theorem, Lebesgue's integrability criteria, composition theorem, product theorem, Integration by parts, Darboux sums, Darboux integrals, Darboux integrability criteria, equivalence of Riemann integral and Darboux integral.

### Unit 2: Sequence and Series of Functions

(35 Marks, 5 Weeks)

Pointwise and uniform convergence of sequence of functions, Theorem on the continuity of the limit function of a sequence of functions, Theorems on the interchange of the limit and derivative, and the interchange of the limit and integrability of a sequence of functions. Pointwise and uniform convergence of series of functions, Theorems on the continuity, Derivability and integrability of the sum function of a series of functions, Cauchy criterion and the Weierstrass M-Test for uniform convergence.

### Unit 3: Improper Integral and Power Series

(30 Marks, 4 weeks)

Improper integrals of Type-I, Type-II and mixed type, Convergence of Beta and Gamma functions, and their properties.

Definition of a power series, Radius of convergence, Absolute convergence (Cauchy-Hadamard theorem), Uniform convergence, Differentiation and integration of power series, Abel's Theorem.

### References:

1. Bartle, Robert G., & Sherbert, Donald R. (2015). *Introduction to Real Analysis* (4th ed.). Wiley India Edition. Delhi.
2. Denlinger, Charles G. (2011). *Elements of Real Analysis*. Jones and Bartlett (Student Edition). First Indian Edition. Reprinted 2015.

3. Ghorpade, Sudhir R. & Limaye, B. V. (2006). *A Course in Calculus and Real Analysis*. Undergraduate Texts in Mathematics, Springer (SIE). First Indian reprint.
4. Ross, Kenneth A. (2013). *Elementary Analysis: The Theory of Calculus* (2nd ed.). Undergraduate Texts in Mathematics, Springer.

**Teaching Plan (MMC-409: Riemann Integration):**

**Week 1:** Definition of Riemann integration.

[1] Chapter 7 [Section (7.1.1 to 7.1.4)]

**Week 2:** Some properties of Riemann integral, Boundedness theorem,

[1] Chapter 7 [Section (7.1.5 to 7.1.7), Exercises of section 7 (1, 2, 7, 8)]

**Week 3:** Riemann integrable function, Cauchy criterion, Squeeze theorem, Riemann integrability of step, continuous, and monotone functions, additive theorem

[1] Chapter 7 [Section (7.2.1 to 7.2.13)]

**Week 4:** Fundamental theorems (First and Second forms), substitution theorem, Lebesgue's integrability criteria, product theorem, Integration by parts

[1] Chapter 7 [Section (7.3.1 to 7.3.17)]

**Week 5:** Darboux sums, Darboux integrals, Darboux integrability criteria, equivalence of Riemann integral and Darboux integral.

[1] Chapter 7 [Section (7.4.1 to 7.4.11)]

**Week 6:** Definitions and examples of pointwise and uniformly convergent sequence of functions.

[1] Chapter 8 [Section 8.1 (8.1.1 to 8.1.10)]

**Week 7:** Motivation for uniform convergence by giving examples. Theorem on the continuity of the limit function of a sequence of functions.

[1] Chapter 8 [Section 8.2 (8.2.1 to 8.2.2)]

**Week 8:** The statement of the theorem on the interchange of the limit function and derivative, and its illustration with the help of examples. The interchange of the limit function and integrability of a sequence of functions.

[1] Chapter 8 [Section 8.2 (Theorems 8.2.3, and 8.2.4)]

**Week 9:** Pointwise and uniform convergence of series of functions, Theorems on the continuity, derivability and integrability of the sum function of a series of functions.

[1] Chapter 9 [Section 9.4 (9.4.1 to 9.4.4)]

**Week 10:** Cauchy criterion for the uniform convergence of series of functions, and the Weierstrass M-Test for uniform convergence.

[2] Chapter 9 [Section 9.4 (9.4.5 to 9.4.6)]

**Week 11:** Improper integrals of Type-I, Type-II and mixed type.

[2] Chapter 7 [Section 7.8 (7.8.1 to 7.8.18)]

**Week 12:** Convergence of Beta and Gamma functions, and their properties.

[3] Pages 405 - 408

**Week 13:** Definition of a power series, Radius of convergence, Absolute and uniform convergence of a power series.

[4] Chapter 4 (Section 23)

**Week 14:** Differentiation and integration of power series, Statement of Abel's Theorem and its illustration with the help of examples.

[4] Chapter 4 [Section 26 (26.1 to 26.6)]

## MMC-410 : Numerical Analysis

**Total Marks:** 100 (Theory: 75 + Internal Assessment: 25)

**Workload:** 6 Lectures (per week),

**Duration:** 14 Weeks (70 hrs ) **Examination:** 3 Hrs.

**Course Objectives:** To comprehend various computational techniques to find approximate value for possible root(s) of non-algebraic equations, to find the approximate solutions of system of linear equations and ordinary differential equations. Also, the use of Computer Algebra System (CAS) by which the numerical problems can be solved both numerically and analytically, and to enhance the problem solving skills.

**Course Learning Outcomes:** The course will enable the students to learn the following:

- i) Some numerical methods to find the zeroes of nonlinear functions of a single variable and solution of a system of linear equations, up to a certain given level of precision.
- ii) Interpolation techniques to compute the values for a tabulated function at points not in the table.
- iii) Applications of numerical differentiation and integration to convert differential equations into difference equations for numerical solutions.

### **Unit 1: Methods for solving Algebraic and Transcendental Equations (30 Marks, 4 weeks)**

Rate of Convergence, Methods of iteration, Bisection method, Newton-Raphson method, Fixed point iteration method, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel method.

### **Unit 2: Interpolation (35 Marks, 5 weeks)**

Finite difference, relation between the operators, ordinary and divided differences, Newton's forward and Backward interpolation formulae, Newton's divided difference formulae and their properties, Lagrange, Hermite and Spline interpolation, Least square polynomial approximation.

### **Unit 3: Numerical Differentiation and Integration (35 Marks, 5 weeks)**

First order and higher order approximation for first derivative, Approximation for second derivative.

Numerical integration by Newton-Cotes formula, Trapezoidal rule, Simpson's rule and its error analysis. Methods to solve ODE's, Picard's method, Euler's and Euler's modified method and Runge-Kutta methods of 2<sup>nd</sup> and 4<sup>th</sup> order.

Solution of boundary value problems of ordinary differential equations using Finite Difference method.

### **References:**

1. Bradie, Brian. (2006). *A Friendly Introduction to Numerical Analysis*. Pearson Education, India. Dorling Kindersley (India) Pvt. Ltd. Third impression 2011.

### **Additional Readings:**

1. Jain, M. K., Iyengar, S. R. K., & Jain, R. K. (2012). *Numerical Methods for Scientific and Engineering Computation*. (6th ed.). New Age International Publisher, India, 2016.
2. Gerald, C. F., & Wheatley, P. O. (2008). *Applied Numerical Analysis* (7th ed.). Pearson Education. India.

**Teaching Plan** (Theory of MMC 410 : Numerical Analysis):

**Week 1:** Algorithms, Convergence, Order of convergence and examples. [1] Chapter 1 (Sections 1.1 and 1.2).

**Week 2:** Bisection method, False position method and their convergence analysis, Stopping condition and algorithms. [1] Chapter 2 (Sections 2.1 and 2.2).

**Week 3:** Fixed point iteration method, its order of convergence and stopping condition. [1] Chapter 2 (Section 2.3).

**Week 4:** Newton's method, Secant method, their order of convergence and convergence analysis. [1] Chapter 2 (Sections 2.4 and 2.5). Department of Mathematics, University of Delhi 49

**Week 5:** Examples to understand partial and scaled partial pivoting. LU decomposition. [1] Chapter 3 (Sections 3.2, and 3.5 up to Example 3.15).

**Weeks 6 and 7:** Application of LU decomposition to solve system of linear equations. Gauss–Jacobi method, Gauss–Seidel. [1] Chapter 3 (Sections 3.5 and 3.8).

**Week 8:** Lagrange interpolation: Linear and higher order interpolation, and error in it. [1] Chapter 5 (Section 5.1).

**Weeks 9 and 10:** Divided difference and Newton interpolation, Piecewise linear interpolation. [1] Chapter 5 (Sections 5.3 and 5.5).

**Weeks 11 and 12:** First and higher order approximation for first derivative and error in the approximation. Second order forward, Backward and central difference approximations for second derivative, Richardson extrapolation method [1] Chapter 6 (Sections 6.2 and 6.3).

**Week 13:** Numerical integration: Trapezoidal rule, Simpson's rule and its error analysis. [1] Chapter 6 (Section 6.4).

**Week 14:** Euler's method to solve ODE's, Second order Runge–Kutta methods: Modified Euler's method, Heun's method and optimal RK2 method. [1] Chapter 7 (Section 7.2 up to Page 562 and Section 7.4, Pages 582-585).

## Semester-V

### MMC-511: Metric Spaces

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorial (per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs,

**Course Objectives:** The course aims at providing the basic knowledge pertaining to metric spaces such as open and closed balls, neighbourhood, interior, closure, subspace, continuity, compactness, connectedness etc.

**Course Learning Outcomes:** After completion of the course, a student will be able to

- i) understand the basic concepts of metric spaces and the concept such as open balls, closed balls
- ii) learn concepts of convergence of sequences, compactness, connectedness and their interrelations
- iii) correlate the concepts of Metric Space with the Analytical concepts such as Continuity and uniform continuity.



**Unit 1: Basic Concepts**

( 30 marks, 4 weeks)

Metric spaces: Definition and examples, Open and closed ball, Neighbourhood, Open set, Interior, exterior, frontier and boundary points of a set, limit point of a set, derived set, closed set, closure of a set, diameter of a set, Dense set, Subspace of a metric space.

**Unit 2: Complete Metric Spaces and Continuous Functions**

( 35 marks, 5 weeks)

Sequences in metric spaces, Cauchy and convergent sequences, Completeness of a metric space, Continuous mappings, Criteria for Continuity, Uniform Continuity, Homeomorphism, Lipschitz Conditions, Contraction mapping, Banach fixed point theorem.

**Unit 3: Connectedness and Compactness**

( 35 marks, 5 weeks)

Connectedness, Components, Connected subsets of  $\mathbb{R}$ , Connectedness and continuity, Compactness, Compactness and Continuity, Sequential compactness, Compactness and finite intersection property, Bolzano-Weierstrass property, Heine-Borel theorem, Totally bounded sets, Compact Subsets of Function Spaces.

**References:**

1. E. T. Copson (1988). *Metric Spaces*. Cambridge University Press.
2. S. Kumaresan (2014). *Topology of Metric Spaces* (2nd ed.). Narosa Publishing House. New Delhi.
3. G.F. Simmons (2004). *Introduction to Topology and Modern Analysis*. Tata McGraw Hill. New Delhi.
4. Satish Shirali & Harikishan L. Vasudeva (2006). *Metric Spaces*. Springer-Verlag.
5. Micheál O'Searcoid (2009). *Metric Spaces*. Springer-Verlag.

**Additional reading**

1. P. K. Jain & Khalil Ahmad (2019). *Metric Spaces*. Narosa Publishing House. New Delhi.

**Teaching Plan ( MMC -511: Metric Spaces)**

**Week 1 :** Metric spaces: Definition and examples[1] Chapter 2, [4] Chapter 1 section 1.2

**Week 2 and 3:** Open and closed ball, Neighbourhood, Open set, Interior, exterior, frontier and boundary points of a set limit point of a set, derived set, closed set, closure of a set, diameter of a set, Dense set, Subspace of a metric space. [1] Chapter 3

**Week 4 and 5:** Sequences in metric spaces, Cauchy and convergent sequences, Completeness of a metric space [1] Chapter 4, [2] Chapter 2

**Week 6 and 7:** Continuous mappings, Criteria for Continuity, Uniform Continuity, Homeomorphism [1] Chapter 7, [4] Chapter 3 section 3.1 – 3.5, [2] Chapter 3.

**Week 8 :** Lipschitz conditions, Contraction mapping, Banach fixed point theorem. [1] Chapter 8, [2] Chapter 6 section 6.4, [4] Chapter 3 section 3.7, [5] Chapter 9.

**Week 9 and 10:** Connectedness, Components, Connected subsets of  $\mathbb{R}$ , Connectedness and continuity [1] Chapter 5, [2] Chapter 5,[4] Chapter 4 section 4.1

**Week 11, 12, 13 and 14:** Compactness, Compactness and Continuity, Sequential compactness, Compactness and finite intersection property, Bolzano-Weierstrass property, Heine-Borel theorem, Totally bounded sets, Compact Subsets of Function Spaces. [1] Chapter 6, [2] Chapter 4, [5] Chapter 12 section 12.8.

## MMC-512 : Mechanics

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorial (per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs,

**Course Objectives:** The course aims at providing the basic knowledge pertaining to dynamics such as simple harmonic motions, particle dynamics, projectiles, Kepler's Law, dynamics of rigid bodies. It also aims to study and impart knowledge on statics such as coplanar forces, catenary, stable and unstable equilibrium.

**Course Learning Outcomes:** This course will enable the students to:

- i) Deal with the kinematics and kinetics of the rectilinear and planar motions of a particle including the constrained oscillatory motions of particles.
- ii) Learn that a particle moving under a central force describes a plane curve and know the Kepler's laws of the planetary motions, which were deduced by him long before the mathematical theory given by Newton.
- iii) Understand necessary conditions for the equilibrium of particles acted upon by various forces and learn the principle of virtual work for a system of coplanar forces acting on a rigid body.

### Unit 1: Dynamics

(35 marks, 5 weeks)

Components of velocities and accelerations along, radial and transverse, along tangential and normal, Simple Harmonic motions, Dynamics of a particle, Motion on smooth and rough plane curves, Motion in resisting medium including projectile, Motion of varying mass, Central orbits and Kepler's Law, Acceleration in different Coordinate system

### Unit 2: Statics

(35 marks, 5 weeks)

Equilibrium condition of coplanar forces, Equilibrium of strings, common catenary, catenary of uniform strength, Force in 3-dimension, Poinso's Central axis, Wrenches Null lines and planes, stable and unstable equilibrium

### Unit 3: Dynamics of Rigid Bodies

(30marks, 4 weeks)

Moments and products of inertia, Momental Ellipsoid, Equipmental systems, Principal Axis, D'Alembert's Principle, Equations of motion of rigid bodies, Motion of centre of inertia, Motion relative to centre of inertia, Motion about a fixed axis, Compound Pendulum, Motion in 2 dimension under finite and impulsive forces, Conservation of momentum and Energy. Euler's dynamical equations for the motion of a rigid body about an axis, Theory of small oscillations.

### References:

- 1 S.L. Loney (1988): *An elementary treatise on dynamics of particle and of rigid bodies.* Cambridge University Press 1956, reprinted by S.Chand & Company (P) Ltd.
- 2 Das & Mukherjee (2010): *Dynamics* published by S. Chand & company (p) Ltd. ISBN-81-85624-96-8.
- 3 Das & Mukherjee (2010): *Statics* published by S.Chand & company (p) Ltd., ISBN-81-85624-18-6.
- 4 S.L. Loney (2004): *An Elementary treatise on Statics* published by A.I.T.B.S., New Delhi, 2004 ISBN-81-7473-123-7.
- 5 A.S. Ramsey (2009): *Statics*, Cambridge University Press

### Additional reading

- 1 M. Ray and G.C. Sharma (2008): *A Textbook of dynamics* published by S. Chand & company (p) Ltd., ( Chapter 1,2,6,8,9,11,12), ISBN-81-219-0342-4.
- 2 R.S. Verma: *A Text Book on Statics*, Pothishala Pvt Ltd., Allahabad.
- 3 A.S. Ramsey (2009): *Dynamics* , Cambridge University Press
- 4 P. L. Srivatava (1964). *Elementary Dynamics*. Ram Narin Lal, Beni Prasad Publishers Allahabad.
- 5 J. L. Synge & B. A. Griffith (1949). *Principles of Mechanics*. McGraw-Hill

### Teaching Plan ( MMC 512: Mechanics)

**Week 1 :** Components of velocities and accelerations along, radial and transverse, along tangential and normal [1] Art 48. 49. 87, 88

**Week 2:** Simple Harmonic motions [1] Art 22-25, [2] Art 17.1 - 17.4. 17.6. 17.7

**Week 3 :** Dynamics of a particle, Motion on smooth and rough plane curves [1]Art 14.1, 14.2, 15.1, 15.2, 16.1, 16.2

**Week 4:** Motion in resisting medium including projectile, Motion of varying mass [1] Art 104-112

**Week 5:** Central orbit, Kepler's Law [1]Art 53-55, 57, 60, 64-67, 69-70), Acceleration in different Coordinate system [1] Art 125-127

**Week 6 and 7:** Equilibrium condition of coplanar forces [3]Art 81, 8.3, Equilibrium of strings, common catenary, catenary of uniform strength [3]Art 14.1-14.5 [5] Art 12.2, 12.21, 12.22, 12.5

**Week 8 and 9 :** Force in 3-dimension, Poinsots Central axis [1]Art 154-157, 162-165,[4] Art 184-186, 188-190

**Week 10:** Wrenches Null lines and planes[4]Art 206-208, stable and unstable equilibrium [4] Art 158, [1]Art 11.5, 11.6, 11.62, 11.7

**Week 11 and 12:** Moments and products of inertia [1]Art 144-149, Momental Ellipsoid [1]Art 151, Equipmental systems, Principal Axis [1]Art 154, 155

**Week 13:** D'Alembert's Principle, Equations of motion of rigid bodies, Motion of centre of inertia, Motion relative to centre of inertia [1]Art 162 . Motion about a fixed axis[1]Art 168 -171, Compound Pendulum [1]Art 173-175

**Week 14:** Motion in 2 dimension under finite and impulsive forces [1]Art 187-190, Conservation of momentum and Energy.[1]Art 235, 236, 238, 239, 242, Euler's dynamical equations for the motion of a rigid body about an axis, Theory of small oscillations.

## Semester VI

### MMC-613: Complex Analysis

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorials (per week)

**Duration:** 14 Weeks (70 Hrs. Theory) **Examination:** 3 Hrs.

**Course Objectives:** This course aims to introduce the basic ideas of analysis for complex functions in complex variables. Particular emphasis has been laid on Cauchy's theorems, series expansions and calculation of residues.

**Course Learning Outcomes:** The completion of the course will enable the students to:

- i) Understand the significance of differentiability of complex functions leading to the understanding of Cauchy-Riemann equations.
- ii) Evaluate the contour integrals and understand the role of Cauchy-Goursat theorem and the Cauchy integral formula.

- iii) Expand some simple functions as their Taylor and Laurent series, classify the nature of singularities, find residues and apply Cauchy Residue theorem to evaluate integrals.

**Unit 1: Analytic Functions and Cauchy-Riemann Equations** (35 marks, 5 weeks)

Functions of complex variables, Limits, Theorems on limits, Limits involving the point at infinity, Continuity, Derivatives, Differentiability, Cauchy-Riemann equations, Sufficient conditions for differentiability; Polar forms of Cauchy-Riemann equations, Analytic functions, Trigonometric function, Multivalued Functions and its branches, Logarithmic functions, Complex exponents.

**Unit 2: Complex Integrals** (30 marks, 4 weeks)

Definite integrals of complex functions over a real interval, Contours, Contour integrals, Antiderivatives, Proof of antiderivative theorem, Cauchy-Goursat theorem, Cauchy integral formula; An extension of Cauchy integral formula, Extension of Cauchy integral formula, Cauchy's inequality, Liouville's theorem and the fundamental theorem of algebra.

**Unit 3: Series and Residues** (35 marks, 5 weeks)

Convergence of sequences and series, Taylor series and its examples; Laurent series and its examples, Absolute and uniform convergence of power series, Uniqueness of series representations, Singular points, Isolated singular points, Residues, Cauchy's residue theorem, residue at infinity; Types of isolated singular points, Residues at poles and its examples.

**References:**

1. H.S. Kasana. (2015). *Complex Variables, Theory and Applications*, (2<sup>nd</sup> ed.). Printice Hall of India Learning Private Limited.
2. Brown, James Ward, & Churchill, Ruel V. (2014). *Complex Variables and Applications* (9th ed.). McGraw-Hill Education. New York

**Additional Readings:**

1. Bak, Joseph & Newman, Donald J. (2010). *Complex analysis* (3rd ed.). Undergraduate Texts in Mathematics, Springer. New York.
2. Zills, Dennis G., & Shanahan, Patrick D. (2003). *A First Course in Complex Analysis with Applications*. Jones & Bartlett Publishers, Inc.
3. Mathews, John H., & Howell, Rusell W. (2012). *Complex Analysis for Mathematics and Engineering* (6th ed.). Jones & Bartlett Learning. Narosa, Delhi. Indian Edition.

**Teaching Plan ( MMC 613: Complex Analysis)**

**Week 1:** Functions of a complex variables, limits, theorems on limits, limits involving the point at infinity, continuity. [1] Chapter 2 (2.1-2.3)

**Week 2:** differentiability, Cauchy- Riemann Equation, sufficient condition for differentiability, Polar form of Chauchy -Riemann Equation. [1] Chapter 2 (2.4-2.5)

**Week 3:** Analytic functions, Exponential, Trigonometric functions. [1] Chapter 2 ( 2.6), Chapter 3 ( 3.1, 3.2, 3.3)

**Week 4:** Multivalued Functions and its branches, Logarithmic functions, Complex Exponents. [1] Chapter 3 (3.5-3.7)

**Week 5:** Definite Integrals of complex functions over a real interval, Contours, Contours Integrals [2] Chapter 4 ( 42-46) , [1] Chapter 4 (4.1-4.2)

**Week 6:** Antiderivatives, Proof of antiderivative theorem, Cauchy-Goursat theorem, [2] Chapter 4 (48-53), [1] Chapter 4 (4.3-4.5)

**Week 7:** Cauchy Integral formula, Extension of Cauchy Integral formula, Consequences of Cauchy Integral formula. [2] Chapter 4 (54-57), [1] Chapter 4 (4.7)

**Week 8 & 9:** Cauchy's inequality, Liouville's theorem and fundamental theorem of algebra. [2] Chapter 4 (57-Theorem 3, 58), [1] Chapter 4 (4.8)

**Week 10:** Convergence of sequences and series, Taylor with examples, [2] Chapter 5 (60-65)

**Week 11:** Laurent series with examples, Absolute and uniform convergence of power series, uniqueness of series representation. [2] Chapter 5 (66-69, 72)

**Week 12 & 13:** Singular points, Isolated singular points, Residues, Residue Theorem, Residue at infinity. [2] Chapter 6 (74-77)

**Week 14:** Types of isolated singular points, Residue at poles with examples. [2] Chapter 6 (78-81)

## MMC-614: Ring Theory & Linear Algebra

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorial (per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The objective of this course is to introduce the fundamental theory of two objects, namely - rings and vector spaces, and their corresponding homomorphisms. It aims to discuss the ring of polynomials and inner product spaces.

**Course Learning Outcomes:** On completion of this course, the students will be able to understand

- i) The fundamental concept of Rings, Fields, subrings, integral domains and the corresponding morphisms.
- ii) The concept of linear independence of vectors over a field, the idea of a finite dimensional vector space, basis of a vector space and the dimension of a vector space.
- iii) Basic concepts of linear transformations, the Rank-Nullity Theorem, matrix of a linear transformation, algebra of transformations, change of basis, eigen values and eigen vectors, orthogonality on vector spaces.

### Unit 1: Rings :

(35 marks, 5 weeks)

Definition and examples of rings, properties of rings, Subrings, Integral Domains & Fields, Characteristics of a ring, Ideals, Ideal generated by a subset of ring, Factor rings, Operations on ideals, Prime Ideal, Principal Ideal and Maximal Ideals, Homomorphism and Isomorphism of Rings, Kernel of a homomorphism; First, Second and Third Isomorphism Theorems, Field of quotients, Polynomial ring over commutative ring, Division algorithm and consequences, Principal ideal domains, Reducibility and Irreducibility tests, Eisenstein's Irreducibility criteria, Unique factorisation in  $\mathbb{Z}[x]$ , Irreducibles, primes, Unique Factorization Domain(UFD), Euclidean domain,

### Unit 2 : Vector Spaces :

(25 marks, 3 weeks)

Concept of vector space over a Field  $K$ , Subspaces, Necessary and sufficient condition for being a Subspace, Algebra of subspaces, Coset of subspace, Quotient Space, Linear combination of vectors, Linear Span, Subspace generated by a subset, Linear dependence & Linear independence, Basis and Dimension with related theorems, Finite Dimensional Vector Space, Dimension of Subspaces, Lagrange Interpolation formula.

**Unit 3: Linear Transformations & Inner Product Space: (40 marks, 6 weeks)**

Linear Transformation, Null space, Ranges, Rank and Nullity of a Linear Transformation Kernel of Linear Transformation, Representation of Linear Transformation as matrices, Algebra of Linear Transformation, Isomorphism and Isomorphism theorem, Dual Space, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilator of a Subspace, Eigenvalues, Eigenvectors, Eigenspaces and Characteristic polynomial of a linear operator, Caley-Hamilton theorem, The minimal polynomial for a linear operator.

Inner product spaces and Norms, Orthonormal basis, Gram-Schmidt Orthogonalization process Orthogonal Complements, Bessel's Inequality for Finite Dimensional Vector Spaces, S

**References:**

1. **Gallian, Joseph. A.** (2013). Contemporary Abstract Algebra (8<sup>th</sup> ed.). Cengage Learning India Private Limited. Delhi
2. **Friedberg, Stephen H, Insel, Arnold J, & Spence Lawrence E.** (2003). *Linear Algebra* (4<sup>th</sup> ed.) Prentice-Hall of India Pvt. Ltd. New Delhi.

**Additional Readings:**

1. **Herstein I.N.** (2006). *Topics in Algebra* (2<sup>nd</sup> ed.). Wiley Student Edition. India
2. **Hoffman, Kenneth, Kunze, Ray Alden** (1978). *Linear Algebra* (2<sup>nd</sup> ed.). Prentice-Hall of India Pvt. Ltd. New Delhi. Pearson Education India Reprint, 2015
3. **V.K.Khanna & S.K.Bhambri:** *A Course in Abstract Algebra*, Vikash Publishing House Pvt Ltd, New Delhi

**Teaching Plan (MMC614: Ring Theory & Linear Algebra)**

**Week 1:** Definition and examples of Ring, properties of Rings, subrings, Integral domain & Fields  
[1] Chapter 12 & 13.

**Week 2&3:** Characteristic of a ring, ideals, ideals generated by a subset of a ring, Factor ring, Operation on ideals; Prime, Maximal & Principal ideals  
[1] Chapter 13 & 14

**Week 4 :** Ring homomorphisms, Properties of ring homomorphisms; First, Second & Third Isomorphism theorems for rings, The field of Quotients.  
[1] Chapter 15 and Exercise 3 & 4 on page 346.

**Week 5:** Polynomial ring over commutative ring, Division algorithm and consequences, Principal ideal domains, Reducibility and Irreducibility tests, Einstein's Irreducibility criterions. Unique factorisation in  $\mathbb{Z}[x]$ , Irreducibles, primes, Unique Factorization Domain(UFD), Euclidean domain.  
[1] Chapter 16 & 17,18

**Week 6:** Concept of vector space over a Field K, Subspaces, Necessary and sufficient condition for being a Subspace, Algebra of subspaces.  
[2] Chapter 1(Sections 1.2 and 1.3)

**Week 7:** Coset of subspace, Quotient Space, Linear combination of vectors, Linear Span, Subspace generated by a subset, Linear dependence & Linear independence.  
[2] Chapter 1 (Section 1.4 and 1.5).

**Week 8:** Basis and Dimension with related theorems, Finite Dimensional Vector Space, Dimension of Subspaces, Lagrange Interpolation formula.  
[2] Chapter 1 (Section 1.6).

**Week 9:** Linear Transformation, Null space, Ranges, Rank and Nullity of a Linear Transformation Kernel of Linear Transformation.  
[2] Chapter 2(section 2.1)

**Week 10:** Representation of Linear Transformation as matrices, Algebra of Linear Transformation, Isomorphism and Isomorphism theorem.  
[2] Chapter 2 (section 2.2, 2.3 & 2.4)

**Week 11:** Dual Space, Double dual, Dual basis, Transpose of a linear transformation and its matrix in the dual basis, Annihilator of a Subspace

[2] Chapter 2 (section 2.6).

**Week 12:** Eigenvalues, Eigenvectors, Eigenspaces and Characteristic polynomial of a linear operator, Caley-Hamilton theorem, The minimal polynomial for a linear operator.

[2] Chapter 5(Section 5.1, 5.2 & 5.4), Chapter 7(Section 7.3, statement of theorem 7.16).

**Week 13 & 14:** Inner product spaces and Norms, Orthonormal basis, Gram-Schmidt Orthogonalization Process, Orthogonal Complements, Bessel's Inequality for Finite Dimensional Vector Spaces.

[2] Chapter 6(sections 6.1 & 6.2)

## Skill Enhancement Paper

### Semester I

#### MMSE-101 A : LaTeX

**Total Marks: 100** (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

**Workload:** 2 Lectures (per week), 4 Practicals (per week per student)

**Duration:** 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

**Course Objectives:** The purpose of this course is to acquaint students with the latest typesetting skills, which shall enable them to prepare high quality typesetting, beamer presentation and webpages.

**Course Learning Outcomes:** After studying this course the student will be able to:

- i) Typeset mathematical formulas, use nested list, tabular & array environments.
- ii) Create or import graphics.
- iii) Use beamer to create presentation .

#### Unit 1: Getting Started with LaTeX(15 marks, 4 weeks)

Introduction to *TeX* and *LaTeX*, Typesetting a simple document, Adding basic information to a document, Environments, Footnotes, Sectioning and displayed material.

#### Unit 2: Mathematical Typesetting with LaTeX(20 marks, 6 weeks)

Accents and symbols, Mathematical Typesetting (Elementary and Advanced): Subscript/ Superscript, Fractions, Roots, Ellipsis, Mathematical Symbols, Arrays, Delimiters, Multiline formulas, Spacing and changing style in math mode.

#### Unit 3: Graphics and Beamer Presentation in LaTeX(15 marks, 4 weeks)

Graphics in LaTeX, Simple pictures using PS Tricks, Plotting of functions, Beamer presentation.

#### References:

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Lamport, Leslie (1994). *LaTeX: A Document Preparation System*, User's Guide and Reference Manual (2nd ed.). Pearson Education. Indian Reprint.

## Practical/Lab work to be performed in Computer Lab.

### Practicals:

[1] Chapter 9 (Exercises 4 to 10), Chapter 10 (Exercises 1 to 4 and 6 to 9),  
Chapter 11 (Exercises 1, 3, 4, and 5), and Chapter 15 (Exercises 5, 6 and 8 to 11).

### Teaching Plan (Theory of MMSE-101A: LaTeX):

**Weeks 1 to 3:** Introduction to TeX and LaTeX, Typesetting a simple document, Adding basic information to a document, Environments, Footnotes, Sectioning and displayed material.

[1] Chapter 9 (9.1 to 9.5)

[2] Chapter 2 (2.1 to 2.5)

**Weeks 4 to 7:** Accents of symbols, Mathematical typesetting (elementary and advanced): subscript/superscript, Fractions, Roots, Ellipsis, Mathematical symbols, Arrays, Delimiters, Multiline formulas, Spacing and changing style in math mode.

[1] Chapter 9 (9.6 and 9.7)

[2] Chapter 3 (3.1 to 3.3)

**Weeks 8 to 10:** Graphics in LaTeX, Simple pictures using PS Tricks, Plotting of functions.

[1] Chapter 9 (Section 9.8)

[1] Chapter 10 (10.1 to 10.3)

[2] Chapter 7 (7.1 and 7.2)

**Weeks 11 to 14:** Beamer presentation.

[1] Chapter 11 (Sections 11.1 to 11.4)

## MMSE-101 B : Computational Mathematics Laboratory

**Total Marks: 100** (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

**Workload:** 2 Lectures (per week), 4 Practicals (per week per student)

**Duration:** 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

**Course Objective:** This course is designed to introduce the student to the basics of power point presentations and working with spread sheets. Also the students of mathematics will have the chance to gain essential skills involving computational mathematics software called mathematica.

**Course Learning Outcomes:** On successful completion of the course, students will be able to

- i). Develop, manage power point presentations while preparing for presentations in seminars with additional skills such as inserting pictures, objects, multimedia etc.
- ii). Work out with excel files with skill of preparing charts to represent the information found in daily life situations.
- iii). Use mathematica software to plot the graph of various functions.

### Unit-1: PowerPoint Presentation

(10 marks, 3 weeks)

Navigate the PowerPoint interface, creating new presentation from scratch – or by using beautiful templates, Add text, Pictures, Sound, Movies and Charts. Designing slides using themes, colours and special effects, Animate objects on slides, work with Master slides to make presentation easy.



**Unit -2: Spreadsheets**

**(15 marks, 4 weeks)**

Examine spreadsheet concepts and explore the Microsoft Office Excel environment, Create, Open and View a workbook. Save and print workbooks. Enter and Edit data. Modify a worksheet and workbook. Work with cell references. Learn to use functions and formulas. Create and edit charts and Graphics. Filter and sort table data. Work with pivot tables and charts. Import and Export data.

**Unit -3: Mathematica**

**(25 marks, 7 weeks)**

Getting Acquainted with the notation and convention, the Kernel and the Front End, Built- functions. Basic operations, Assignment and Replacement. Logical Relations, Sum and Products, Loops.

Two Dimensional Graphics – plotting functions of a single variable, Additional Graphics Commands, Animations.

Three Dimensional Graphics – plotting functions of two variables, Special three dimensional plots.

Equation(s) solving commands, Matrix operations – vectors and matrices operations, eigenvalues and eigenvectors, trace, adjoint, inverse, diagonalization etc.

**References:**

1. Binder, Donald & Erickson, Martin (2011). A student's guide to the Study, Practice, and Tools of Modern Mathematics. CRC Press, Taylor & Francis Group, LLC.
2. Hillier and Hillier (2003). Introduction to Management Science: A Modeling and Case Studies Approach with Spreadsheet, Second Edition, McGraw-Hill.
3. Eugene Don, Ph. D., Schaum's Outlines Mathematica, Mc-Graw Hill (2009).

**List of Practical to be performed at the Laboratory:**

**a) PowerPoint Presentation:**

1. Change the fonts, colour of text on a slide
2. Add bullets or numbers to text
3. Format text as superscript or subscript
4. Insert a picture that is save on your local drive or an internal server
5. Insert a picture from the web
6. Insert shapes in your slide

**b) Spreadsheet:**

1. Format, enhance, and insert formulas in spreadsheet.
2. Move data within and between workbooks.
3. Maintain a workbook and create a chart in a spreadsheet.
4. Create, modify and manage a database table and query.
5. Create relationships between tables in a database.
6. Import and export data among word processing software, a spreadsheet and a database.
7. Merge data in a database with a word processing document.

c) **Mathematica:**

1. In an expression containing  $x, y, z$  replace all  $x, y, z$  by  $x^2, y^2$  and  $z^2$ .
2. Find the sum of i)  $1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{100}$ , ii)  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$  to  $\infty$
3. Solve the equation i)  $x^3 - x + 1 = 0$  for  $x$ , ii) Solve:  $x - y = 1, x^2 - xy + y^2 = 10$
4. Plot the graph of  $\sin x$  and  $\cos x$  together, where  $-\pi \leq x \leq \pi$
6. Plot the graph of the functions  $\sin \pi x \sin \pi y$ , where  $-1 \leq x \leq 1$  and  $-1 \leq y \leq 1$

## Semester II

### MMSE-202 A : Python Programming

**Total Marks: 100** (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

**Workload:** 2 Lectures (per week), 4 Practicals (per week per student)

**Duration:** 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

**Course Objective:** This course is designed to introduce the student to the basics of programming using Python. The course covers the topics essential for developing well documented modular programs using different instructions and built-in data structures available in Python.

**Course Learning Outcomes:** On successful completion of the course, students will be able to

1. Develop, document, and debug modular python programs to solve computational problems.
2. Select a suitable programming construct and data structure for a situation.
3. Use built-in strings, lists, sets, tuples and dictionary in applications.
4. Define classes and use them in applications.
5. Use files for I/O operations.

#### Unit 1 Introduction to Programming using Python

(20 marks, 6 weeks)

Structure of a Python Program, Functions, Interpreter shell, Indentation. Identifiers and keywords, Literals, Strings, Basic operators (Arithmetic operator, Relational operator, Logical or Boolean operator, Assignment Operator, Bit wise operator). Building blocks of Python: Standard libraries in Python, notion of class, object and method.

#### Unit 2 Creating Python Programs

(15 marks, 4 weeks)

Input and Output Statements, Control statements: -branching, looping, Exit function, break, continue and pass, mutable and immutable structures. Testing and debugging a program.

#### Unit 3 Visualization using 2D and 3D graphics and data structures

(15 marks, 4 weeks)

Visualization using graphical objects like Point, Line, Histogram, Sine and Cosine Curve, 3D objects, Built-in data structures: Strings, lists, Sets, Tuples and Dictionary and associated operations. Basic searching and sorting methods using iteration and recursion.

**References:**

1. Downey, A.B., (2015),*Think Python–How to think like a Computer Scientist*, 3<sup>rd</sup> edition. O’Reilly Media.
2. Taneja, S. &Kumar,N., (2017),*Python Programming-A Modular Approach*. Pearson Education.

**Additional Reading:**

1. Brown, M. C. (2001).*TheComplete Reference:Python*, McGraw Hill Education.
2. Dromey, R. G. (2006),*How to Solve it by Computer*, Pearson Education.
3. Guttag, J.V.(2016),*Introduction to computation and programming using Python*. MIT Press.
- 4.Liang,Y.D. (2013),*Introduction to programming using Python*. PearsonEducation.

**Practical**

1. Execution of expressions involving arithmetic, relational, logical, and bitwise operators in the shell window of Python IDLE.
2. Write a Python function to produce the outputs such as:

a)

```
      *
     ***
    *****
   ***
  *

```

(b)

```
1
232
34543
4567654
567898765

```

3. Write a Python program to illustrate the various functions of the “Math”module.
4. Write a function that takes the lengths of three sides:**side1**, **side2** and **side3** of the triangle as the input from the user using **input** function and return the area of the triangle as the output. Also, assert that sum of the length of any two sides is greater than the third side.
5. Consider a showroom of electronic products, where there are various salesmen. Each salesman is given a commission of 5%, depending on the sales made per month. In case the sale done is less than 50000, then the sales man is not given any commission. Write a function to calculate total sales of a salesman in a month, commission and remarks forthe salesman. Sales done by each salesman per week is to be provided as input. Assign remarks according to the following criteria:  
Excellent: Sales >=80000  
Good: Sales>=60000 and <80000  
Average: Sales>=40000 and <60000  
Work Hard: Sales <40000
6. Write a Python function that takes a number as an input from the user and computes its factorial.

7. Write a Python function to return nth terms of Fibonacci sequence
8. Write a function that takes a number with two or more digits as an input and finds its reverse and computes the sum of its digits.
9. Write a function that takes two numbers as input parameters and returns their least common multiple and highest common factor.
10. Write a function that takes a number as an input and determine whether it is prime or not.
11. Write a function that finds the sum of then terms of the following series:
  - a)  $1 - x^2 / 2! + x^4 / 4! - x^6 / 6! + \dots x^n / n!$
  - b)  $1 + x^2 / 2! + x^4 / 4! + x^6 / 6! + \dots x^n / n!$
12. Write a Python function that takes two strings as an input from the user and counts the number of matching characters in the given pair of strings.
13. Write a Python function that takes a string as an input from the user and displays its reverse.
14. Write a Python function that takes a string as an input from the user and determines whether it is palindrome or not.
15. Write a Python function to calculate the sum and product of two compatible matrices
16. Write a function that takes a list of numbers as input from the user and produces the corresponding cumulative list where each element in the list present at index i is the sum of elements at index  $j \leq i$ .
17. Write a function that takes **n** as an input and creates a list of n lists such that  $i^{\text{th}}$  list contains first five multiples of i.
18. Write a function that takes a sentence as input from the user and calculates the frequency of each letter. Use a variable of dictionary type to maintain the count.
19. Write a Python function that takes a dictionary of *word:meaning* pairs as an input from the user and creates an inverted dictionary of the form meaning:list-of-words.
20. Usage of Python debugger tool-pydbandPython Tutor.
21. Implementation of Linear and binary search techniques
22. Implementation of selection sort, insertion sort, and bubble sort techniques
23. Write a menu-driven program to create mathematical 3D objects Curve, Sphere, Cone, Arrow, Ring, and Cylinder.
24. Write a program that makes use of a function to accept a list of n integers and displays a histogram.
25. Write a program that makes use of a function to display sine, cosine, polynomial and exponential curves.
26. Write a program that makes use of a function to plot a graph of people with pulse rate p vs. height h. The values of p and h are to be entered by the user.
27. Write a function that reads a file **file1** and displays the number of words and the number of vowels in the file.
28. Write a Python function that copies the content of one file to another.
29. Write a function that reads a file **file1** and copies only alternative lines to another file **file2**. Alternative lines copied should be the odd numbered lines.

### Teaching Plan (MMSE-202A Python Programming )

**Week 1:** Python Programming: An Introduction Structure of a Python program, understanding Python interpreter/Python shell, indentation. Atoms, identifiers and keywords, literals, Python strings, arithmetic operator, relational operator, logical or Boolean operator, bit wise operators.

**Week 2:** Variables and Functions Python standard libraries such as sys and math. Variables and assignment statements. Built-in functions such as input and print.

**Week 3-4:** Control Structures if conditional statement and for loop, While loop, break, continue, and pass statement, else statement. Infinite loop

- Week 5:** Functions Function definition and call, default parameter values, keyword arguments, assert statement
- Week 6:** Strings and Lists Strings-slicing, membership, and built-in functions on strings Lists- list operations.
- Week 7:** Mutable object Lists- built-in functions, list comprehension, passing list as arguments, copying list objects.
- Week 8:** Sets, tuples, and dictionary- associated operations and built-in functions.
- Week 9:** Testing and Debugging Determining test cases, use of python debugger tool- pydb for debugging
- Week 10:** Searching and Sorting Linear search, binary search, selection sort, insertion sort, and bubble sort
- Week 11:** Python 2D and 3D Graphics Visualization using graphical objects like point, line, histogram, sine and cosine curve, 3D objects
- Week 12:** File Handling Reading and writing text and structured files.
- Week 13:** Errors and Exceptions Types of errors and exceptions, and exception handling
- Week 14:** Classes Notion of class, object, and method.

## **MMSE-202 B: Computer Algebra Systems and Related Software**

**Total Marks: 100** (Theory: 37.5, Internal Assessment: 12.5 and Practical: 50)

**Workload:** 2 Lectures (per week), 4 Practicals (per week per student)

**Duration:** 14 Weeks (28 Hrs. Theory + 56 Hrs. Practical) **Examination:** 2 Hrs.

**Course Objectives:** This course aims at familiarizing students with the usage of computer algebra systems (/Mathematica/MATLAB/Maxima/Maple) and the statistical software **R**. The basic emphasis is on plotting and working with matrices using CAS. Data entry and summary commands will be studied in **R**. Graphical representation of data shall also be explored.

**Course Learning Outcomes:** This course will enable the students to:

- i) Use CAS as a calculator, for plotting functions, animations and various applications of matrices.
- ii) Understand the use of the software **R** for entry, summary calculation, pictorial representation of data and exploring relationship between data.
- iii) Analyze, test, and interpret technical arguments on the basis of geometry.

### **Unit 1: Introduction to CAS and Applications (15 marks, 4 weeks)**

Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Plotting functions of two variables using Plot3D and ContourPlot, Plotting parametric curves surfaces, Customizing plots, Animating plots, Producing tables of values, working with piecewise defined functions, Combining graphics.

### **Unit 2: Working with Matrices(15 marks, 4 weeks)**

Simple programming in a CAS, Working with matrices, Performing Gauss elimination, operations (transpose, determinant, inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, eigenvector and diagonalization.

### Unit 3: R - The Statistical Programming Language (20 marks, 6 weeks)

**R** as a calculator, Explore data and relationships in **R**. Reading and getting data into **R**: Combine and scan commands, Types and structure of data items with their properties. Manipulating vectors, Data frames, Matrices and lists. Viewing objects within objects. Constructing data objects and conversions, Summary commands: Summary statistics for vectors, Data frames, Matrices and lists. Summary tables. Stem and leaf plot, Histograms. Plotting in **R**: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and bar charts. Copy and save graphics to other applications.

#### References:

1. Bindner, Donald & Erickson, Martin. (2011). *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. CRC Press, Taylor & Francis Group, LLC.
2. Torrence, Bruce F., & Torrence, Eve A. (2009). *The Student's Introduction to Mathematica®: A Handbook for Precalculus, Calculus, and Linear Algebra* (2nd ed.). Cambridge University Press.
3. Gardener, M. (2012). *Beginning R: The Statistical Programming Language*, Wiley.

**Note:** Theoretical and Practical demonstration should be carried out only in **one** of the CAS: Mathematica/MATLAB/Maxima/Scilab or any other.

### Practical/Lab work to be performed in Computer Lab.

#### Practicals:

- [1] Chapter 12 (Exercises 1 to 4 and 8 to 12), Chapter 14 (Exercises 1 to 3)
- [2] Chapter 3 [Exercises 3.2 (1 and 2), 3.3 (1, 2 and 4), 3.4 (1 and 2), 3.5 (1 to 4), 3.6 (2 and 3)].
- [2] Chapter 6 (Exercises 6.2 and 6.3).
- [2] Chapter 7 [Exercises 7.1 (1), 7.2, 7.3 (2), 7.4 (1) and 7.6].

**Note:** Relevant exercises of [3] Chapters 2 to 5 and 7 (The practical may be done on the database to be downloaded from <http://data.gov.in/>).

#### Teaching Plan (Theory of MMSE-202B: Computer Algebra Systems and Related Software):

**Weeks 1 to 3:** Computer Algebra System (CAS), Use of a CAS as a calculator, Computing and plotting functions in 2D, Producing tables of values, Working with piecewise defined functions, Combining graphics. Simple programming in a CAS. [1] Chapter 12 (Sections 12.1 to 12.5). [2] Chapter 1, and Chapter 3 (Sections 3.1 to 3.6 and 3.8).

**Weeks 4 and 5:** Plotting functions of two variables using Plot3D and contour plot, Plotting parametric curves surfaces, Customizing plots, Animating plots. [2] Chapter 6 (Sections 6.2 and 6.3). **Weeks 6 to 8:** Working with matrices, Performing Gauss elimination, Operations (Transpose, Determinant, Inverse), Minors and cofactors, Working with large matrices, Solving system of linear equations, Rank and nullity of a matrix, Eigenvalue, Eigenvector and diagonalization. [2] Chapter 7 (Sections 7.1 to 7.8).

**Weeks 9 to 11:** R as a calculator, Explore data and relationships in R. Reading and getting data into R: Combine and scan commands, Types and structure of data items with their properties. Manipulating vectors, Data frames, Matrices and lists. Viewing objects within objects. Constructing data objects and conversions. [1] Chapter 14 (Sections 14.1 to 14.4). [3] Chapter 2, and Chapter 3. **Weeks 12 to 14:** Summary commands: Summary statistics for vectors, Data frames, Matrices and lists. Summary tables. Stem and leaf plot, histograms. Plotting in R: Box-whisker plots, Scatter plots, Pairs plots, Line charts, Pie charts, Cleveland dot charts and Bar charts. Copy and save graphics to

## Generic Elective (GE) Course -Mathematics Semester-III

### MMGE-301: QUANTITATIVE APTITUDE

**Total Marks: 100** (Theory: 70, Internal Assessment: 30)

**Workload: 6** Lectures, (per week)

**Duration: 14 Weeks (70 Hrs.) Examination: 3 Hrs.**

**Course Objectives:** The main aim of this course is to gain knowledge of elementary ideas about arithmetic abilities which one finds in daily life. It will help the students from any background to get acquainted with this knowledge and get prepared for any competitive examinations.

**Course Learning Outcomes:** This course will enable the students to:

- i) gain sufficient ideas of mental and arithmetic abilities.
- ii) handle mental/quantitative aptitude test questions with great ease.
- iii) acquire the skill of solving problems of daily life quickly.

#### Unit-1: Arithmetic Ability I

(30 marks, 4 weeks)

Chain Rule –Time and Work – Pipes and Cisterns

Time and Distance – Problems on Trains – Boats and Streams

#### Unit-2: Arithmetic Ability II

(30 marks, 4 weeks)

Simple Interest – Compound Interest – Stocks and Shares. (Chapters 17, 18 & 19)

Clocks – Area (Chapters 24, 25)

#### Unit-3: Arithmetic Ability III

(40 marks, 6 weeks)

Volume and Surface Area. (Chapters 28)

Permutations and Combinations. (Chapters 30 & 31)

#### Text Book:

1. Scope and treatment as in “Quantitative Aptitude”, S. Chand and Company Ltd. Ram Nagar, New Delhi (2007).

#### Teaching plan (MMGE- 301: Quantitative Aptitude):

**Week 1& 2:**Chain Rule –Time and Work – Pipes and Cisterns,[1] Chapters 14, 15 & 16.

**Week 3 &4 :**Time and Distance – Problems on Trains – Boats and Streams [1] Chapters 21, 22 & 29.

**Week 5 &6:**Simple Interest – Compound Interest – Stocks and Shares. [1]Chapters 17, 18 & 19.

**Week 7 & 8:**Clocks – Area [1] Chapters 24, 25.

**Week 9:**Volume and Surface Area. [1] Chapter 28.

**Week 10 to 14:**Permutations and Combinations. [1] Chapters 30 & 31.

## Semester-IV

### MMGE-402 : BASIC TOOLS OF MATHEMATICS

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 6 Lectures(per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The objective of the course is to introduce the concept of geometry, vectors algebraic ideas like various forms of mean, progression, polynomial will be taught to the students. The concept of differential calculus and probability will help the students in understanding their respective core courses with great comfort.

**Course learning Outcomes:** After studying this course, the student may understand

1. The basic concepts of Geometry and Vectors Analysis.
2. Some topics of Algebra and Differential Calculus.
3. Application of partial differentiation in daily life problems.
4. Properties and methods of Integration, solving of definite and indefinite integrals.
5. Basic ideas of probability such as probability distribution, expectations, Binomial Distribution, Poisson distribution, etc.

#### UNIT-1: Geometry and Vectors:

(40 marks, 6 weeks)

##### Geometry

Three Dimensional space, Rectangular Cartesian Coordinates, Polar Coordinates, Cylindrical Coordinates, Spherical coordinates. Change of origin, Section of a line joining two given points.

##### Vectors

Addition of two or more vectors, Negative of a vector, , Subtraction of two vectors, Multiplication of a vector by a scalar, Vector equations, Collinear vectors, Position vector of a point, Section Ratio of a point, Linear combination of a set of vectors, Coordinates of two and three dimensional vectors. Product of two or three vectors.

#### UNIT-2: Algebra and Calculus

(40 marks, 6 weeks)

##### Algebra

Geometric Mean, Arithmetic Mean, Harmonic Mean and related Inequalities, Arithmetic and Geometric Progression, Polynomial, Equation, Linear Equation, Quadratic Equation, Roots and Coefficients, Fundamental Theorem of Algebra, Binomial Theorem, Permutation, and Combination, Mathematical Induction, Determinants, Matrices, Solution of equations by matrix method.

##### Differential Calculus

Mappings, Inverse Mapping and Composite Mappings.

Limit, Continuity, Differentiation, Maxima and Minima, Tangent and normal, Partial Differentiation.

##### Integral Calculus

Definition, Properties, Methods of Integration, Definite integrals, Infinite Integrals.



**UNIT-3: Probability**

(20 marks, 2 weeks)

**Probability**

Definition, Random variable (discrete and continuous), Probability Distribution (mass function, density function, distribution function), Expectations, Some Standard Probability Distributions (Distributions: Binomial, Poisson, Negative Binomial, Geometric, Hyper-geometric, Normal, Exponent, Uniform, Gamma, Beta, etc.)

**Recommended books**

1. B.S.Vatssa: Discrete Mathematics ch.1, 2e, WishwaPrakashan (A Division of Wiley Eastern Ltd.)
2. Chandrika Prasad: Algebra and Theory of Equations, Pothisala Pvt. Ltd.
3. Das and Mukherjee: Differential Calculus, UN Dhur& Sons Pvt. Ltd.
4. Das and Mukherjee: Integral Calculus, UN Dhur& Sons Pvt. Ltd.
5. Ghosh&Maity: Vector Analysis, New Central Book Agency, Kolkata
6. S.C. Gupta and V.K. Kapoor: Fundamentals of Mathematical Statistics, Sultan Chand & Sons.
7. Chakraborty&Ghosh: Analytical Geometry and Vector Analysis, UN.Dhur& Sons, Kolkata
8. Chakraborty&Ghosh: Advanced Analytical Geometry , UN.Dhur& Sons, Kolkata

**Semester - V**

**MMGE –503 : RECREATIONAL MATHEMATICS**

**Total Marks:** 100 (Theory: 75, Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1Tutorial (per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The main objective of this course is to impart the knowledge of Mathematics used in daily life to the students with little background in the subject and to provoke enthusiasm.

**Course Learning Outcomes(s)**

After studying this course, the students will be able

1. To understand basic set theory, mathematical puzzles, beauty of figurate numbers and to solve real-life problems.
2. To understated CRT, Fermat's Little Theorem, Euler's Theorem, Wilson's Theorem, application of congruences, application of Mathematics in Nature, Geometric shapes, patterns, etc.
3. To understand the application of Number Theory in ISSN, ISBN, UPC, Credit card check and have a knowledge about some mathematicians viz, Ramanujan, Hardy, Erdos etxc.

**Unit-1: Basic Set Theory and Fundamentals**

**(30 marks, 4 weeks)**

Notations, Venn Diagram, Union, Intersection, Complement, Comparable, sets of Numbers, Line Diagram of the Number System, Intervals, Algebra of Sets.

Order Relation, Absolute Value, Summation Notation, Indexed Summation, Product Notation, Well Ordering Principle.

Recursion, Handshake Problem, Tower of Brahma, Binomial Theorem, Pascal's Identity, Pascal's Triangle, Magic Squares, Geometrical Patterns.

Polygonal, Triangular, Square, Pentagonal, Hexagonal, Pyramidal. Triangular Pyramidal, Square Pyramidal, Pentagonal Pyramidal, Hexagonal Pyramidal numbers.

**Unit 2: Congruences**

**(35 marks, 5 weeks)**

Basic properties of congruences, congruence classes, linear congruence, solutions, Chinese Remainder Theorem, Some special theorems, Fermat's little theorem, Euler's theorem, Wilson's theorem, Application of congruence; Divisibility test, check digits.

Detection of error in an ISBN, ISSN, product code(UPC), credit card check digit, application of congruences in sports, setting time table for tournaments.

**Unit 3: Some applications and Biography of some Mathematicians**

**(35 marks, 5 weeks)**

Palindromic number, Taxicab number (Hardy-Ramanujan Number).

Pythagorean Triples, Pythagorean triples and the unit circle.

Fibonacci Numbers, Fibonacci sequence, Fibonacci Problem, Dynamical Growth of rabbit population and Fibonacci sequence, Some Fascinating Numbers of Lucas, Examples of Mathematics in Nature, Geometric shapes, Symmetry, Fibonacci Spiral, Golden Ratio, Fractals.

Historical Notes on S. Ramanujan, G. Hardy, Paul Erdos, Aryabhata, Brahmagupta, Bhaskara.

**References:**

1. Seymour Lipschutz, Set Theory and Related Topics, Schaum's Outline Series, TMH/McGraw Hill
2. Thomas Koshy (2007): Elementary Number Theory with Applications, (Second Edn.), Elsevier.
3. Joseph H. Silverman (2014): A Friendly Introduction to Number Theory, (Fourth Edn), Pearson IN.
4. Neville Robbins: Beginning Number Theory, (Second Edn.), Jones & Barlett Learning.
5. M.K.Sen, B.C.Chakraborty (2002): Introduction to Discrete Mathematics, NCBA Publishers.
6. Wikipedia

**Teaching Plan (MMGE – 503: Recreational Mathematics)**

**Week 1:** Venn Diagram, Union, Intersection, Complement, Comparable, sets of Numbers, Line Diagram of the Number System, Intervals, Algebra of Sets. ([1] Ch.1,2, 3, 7)

**Week 2:** Order Relation, Absolute Value, Summation Notation, Indexed Summation, Product Notation, Well Ordering Principle ([2] Ch. 1 sec 1.1 to 1.3)

**Week 3:** Recursion, Handshake Problem, Tower of Brahma, Binomial Theorem, Pascal's Identity, Pascal's Triangle, Magic Squares, Geometrical Patterns ([2] Ch. 1 sec 1.4, 1.5).

**Week 4:** Polygonal, Triangular, Square, Pentagonal, Hexagonal, Pyramidal. Triangular Pyramidal, Square Pyramidal, Pentagonal Pyramidal, Hexagonal Pyramidal ([2] Ch.1 Sec 1.6, 1.7)

**Week 5:** Congruences, congruence classes, linear congruence, solutions, Chinese Remainder Theorem ([2] Ch. 4, [4] Ch. 4 Sec. 4.2, 4.3)

**Week 6 and 7:** Chinese Remainder Theorem, Some special theorems, Fermat's little theorem, Euler's theorem, Wilson's theorem. ([2] Ch. 6, 7, [5] Ch. 3)

**Week 8 and 9:** Application of congruence; Divisibility test, check digits, ISBN, ISSN, product code(UPC), credit card check digit, application of congruences in sports, setting time table for tournaments ([2] Ch.5 sec 5.1, 5.3,5.5, [5] Ch.4)

**Week 10:** Palindromic number, Taxicab number (Hardy-Ramanujan Number) [6], Pythagorean Triples, Pythagorean triples and the unit circle ([3] Ch. 2 & 3), [6].

**Week 11, 12 and 13:** Fibonacci Numbers, Fibonacci sequence, Fibonacci Problem, Dynamical Growth of rabbit population and Fibonacci sequence, Some Fascinating Numbers of Lucas, Examples of Mathematics in Nature, Geometric shapes, Symmetry, Fibonacci Spiral, Golden Ratio, Fractals ([2] Ch.2 Sec 2.6, ..) [6]

**Week 14:** Historical Notes on S. Ramanujan, G. Hardy, Paul Erdos, Aryabhata, Brahmagupta, Bhaskara [6].

## **Semester - VI**

### **MMGE-604: Discrete Mathematics**

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 5 Lectures, 1 Tutorial (per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The course introduces formal logic notation, methods of proof, mathematical induction, set theory, permutations and combinations and counting principles. One can learn the concepts of lattices and Boolean algebra in analysis of various applications.

**Course Learning Outcomes:** This course will enable the students to:

- i) Understand the basic principles of logic, set theory, lattices and Boolean algebra.
- ii) Understand the ideas of basic counting techniques.
- iii) Proficiently construct logical arguments and rigorous proofs.

**Unit 1: Logical Mathematics**

**(30 marks, 4 weeks)**

Compound statements (and, or, implication, negation, contrapositive, quantifiers), Proofs in Mathematics, Truth tables, Basic logical equivalences and its consequences, Logical arguments, Binary relations, Types of binary relations, Equivalence relations, Partial and total ordering (Hasse diagram, Lexicographic order, Isomorphism, extremal elements).

**Unit 2: Lattices and its Properties**

**(35 marks, 5 weeks)**

Lattices, Duality principle, Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices, Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn–McCluskey method, Karnaugh diagrams, Switching circuits and applications of switching circuits.

**Unit 3: Applications of Numbers**

**(35 marks, 5 weeks)**

Properties of integers, Division algorithm, Divisibility and Euclidean algorithm, GCD, LCM, Relatively prime, Prime numbers, Statement of fundamental theorem of arithmetic, Fermat primes, Recursively defined sequences, Recursive relations and its solution (characteristics polynomial and generating function), Principles of counting (Inclusion/Exclusion, Addition and Multiplication rule, Pigeon-Hole).

**References:**

1. Bernard Kolman, Robert C Busby, Sharon C Ross (2004). *Discrete Mathematical Structures*. (Fifth Edition) Pearson Education, Inc..
2. Goodaire, Edgar G., & Parmenter, Michael M. (2005). *Discrete Mathematics with Graph Theory* (3rd ed.). Pearson Education (Singapore) Pvt. Ltd. Indian Reprint.
3. Lidl, Rudolf & Pilz, Günter. (1998). *Applied Abstract Algebra* (2nd ed.). Undergraduate Texts in Mathematics. Springer (SIE). Indian Reprint 2004.

**Additional Reading:**

1. Rosen, Kenneth H. (2012) *Discrete Mathematics and its Applications* (7th ed.). McGraw-Hill Education (India) Pvt. Ltd.

**Teaching Plan (MMGE-604): Discrete Mathematics**

**Week 1:** Compound statements (and, or, implication, negation, contrapositive, quantifiers), Proofs in Mathematics

[2] Chapter 1 (section 1.1, 1.2)

**Week 2:** Truth tables, Basic logical equivalences and its consequences

[2] Chapter 1(section 1.3, 1.4)

**Week 3:** Logical arguments, Binary relations

[2] Chapter 1(section 1.5) Chapter 2(section 2.3)

**Week 4:** Types of binary relations, Equivalence relations, Partial and total ordering, (Hasse diagram, Lexicographic order, Isomorphism, extremal elements)

[2] Chapter 2(section 2.4, 2.5), [1] Chapter 6(section 6.1, 6.2)

**Week 5 & 6:** Lattices, Duality principle, Lattices as ordered sets, Lattices as algebraic structures, Sublattices, Products and homomorphisms, Distributive lattices

[1] Chapter 6 (section 6.3), [3] Chapter 1(section 1& 2)

**Week 7 & 8:** Boolean algebras, Boolean polynomials, Minimal forms of Boolean polynomials, Quinn–McCluskey method, Karnaugh diagrams,  
[3] Chapter 1(section 3, 4, 6) [1] Chapter 6 (section 6.4 , 6.5, 6.6)

**Week 9:** Switching circuits and applications of switching circuits.  
[3] Chapter 2(section 7 & 8)

**Week 10:** Properties of integers, Division algorithm,  
[2] Chapter 4(section 4.1 to 4.1.6)

**Week 11:** Divisibility and Euclidean algorithm, GCD, LCM, Relatively prime  
[2] Chapter 4(section 4.2)

**Week 12:** Prime numbers, Statement of fundamental theorem of arithmetic, Fermat primes,  
[2] Chapter 4(section 4.3)

**Week 13:** Recursive relations and its solution (characteristics polynomial and generating function),  
[2] Chapter 5(section 5.2, 5.3, 5.4)

**Week 14:** Principles of counting (Inclusion/Exclusion, Addition and Multiplication rule, Pigeon-Hole)  
[2] Chapter 6 (section 6.1, 6.2, 6.3)

## **DISCIPLINE SPECIFIC ELECTIVE COURSE (DSE)**

### **MME : 501 [SEMESTER – V]**

#### **ANY ONE OF THE FOLLOWING ADVANCED GROUP THEORY/ MATHEMATICAL MODELING/INTEGRAL TRANSFORM**

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorial (per week )

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

### **MME-501 A : Advanced Group Theory**

**Course Objectives:** The course will develop an in-depth understanding of one of the most important branch of the abstract algebra with applications to practical real-world problems. Classification of all finite Abelian groups (up to isomorphism) can be done.

**Course Learning Outcomes:** The course shall enable students to learn about:

- i) Automorphisms for constructing new groups from the given group.
- ii) External direct product that applies to data security and electric circuits.
- iii) Group actions, Sylow theorems and their applications to check nonsimplicity.

**Unit 1: Automorphisms and External and Internal Direct Products (35 marks, 5 weeks)**

Automorphism, inner automorphism, Automorphism groups, Automorphism groups of finite and infinite cyclic groups, Characteristic subgroups, Commutator subgroup and its properties; Applications of factor groups to automorphism groups.

External direct products of groups and its properties, The group of units modulo  $n$  as an external direct product, Applications to data security and electric circuits; Internal direct products, Fundamental theorem of finite Abelian groups and its isomorphism classes.

**Unit 2: Group Action (30 marks, 4 weeks)**

Group actions and permutation representations; Stabilizers, orbits and kernels of group actions; Groups acting on themselves by left multiplication and consequences; Conjugacy in  $S_n$ , Conjugacy classes, The class equation,  $p$ -groups.

**Unit 3: Sylow Theorems and applications (35 marks, 5 weeks)**

The Sylow theorems and consequences, Applications of Sylow theorems; Classification of groups of order  $p^2$ , where  $p$  is a prime, Classification of groups of  $pq$  where  $p, q$  are distinct primes; Finite simple groups, Non simplicity tests; Generalized Cayley's theorem, Index theorem, Embedding theorem and applications. Simplicity of  $A_5$ .

**References:**

1. Dummit, David S., & Foote, Richard M. (2016). *Abstract Algebra* (3<sup>rd</sup> ed.). Student Edition. Wiley India.
2. Gallian, Joseph. A. (2013). *Contemporary Abstract Algebra* (8<sup>th</sup> ed.). Cengage Learning India Private Limited. Delhi.
3. Herstein, I. N. (2006.) *Topics in Algebra* (2<sup>nd</sup> Edition). Wiley India.
4. Michael Artin (2014). *Algebra* (2<sup>nd</sup> Edition). Pearson.
5. Bhattacharya, P.B., Jain, S.K. and Nagpaul, S.R. (2003). *Basic Abstract Algebra* (2<sup>nd</sup> Edition). Cambridge University Press.

**Additional Reading:**

1. Rotman, Joseph J. (1995). *An Introduction to The Theory of Groups* (4<sup>th</sup> Edn.) Springer-Verlag, New York.
2. Khanna, Vijay K, Bhambri, S K (2017). *A Course in Abstract Algebra*(5<sup>th</sup> edition). Vikas Publishing House.

**Teaching Plan ( Theory of MME -501(A): Advanced Group Theory)**

**Week 1:** Automorphism, inner automorphism, Automorphism groups, Automorphism groups of finite and infinite cyclic groups

[1] Chapter 4(section 4.4), [ 2] Chapter 6 (page 135-139)

**Week 2:** Characteristic subgroups, Commutator subgroup and its properties; Applications of factor groups to automorphism groups,

[1] Chapter 4(section 4.4), [ 2] Chapter 9(Theorem 9.4)

**Week 3:** External direct products of groups and its properties, The group of units modulo  $n$  as an external direct product, Applications to data security and electric circuits,

[ 2] Chapter 8

**Week 4:** Internal direct products,

[ 2] Chapter 9( section on internal direct product)

**Week 5:** Fundamental theorem of finite Abelian groups and its isomorphism classes.

[ 2] Chapter 11, [1]Chapter 5(section 5.2), [5] Chapter 8

**Week 6 and 7:** Group actions and permutation representations; Stabilizers and kernels of group actions

[1] Chapter 1(section 1.7), Chapter 2(section2.2), Chapter 4(section4.1)

**Week 8:** Groups acting on themselves by left multiplication and consequences;

[1] Chapter 4(section 4.2 and 4.3) Conjugacy in  $S_n$

**Week 9:** Conjugacy classes, The class equation,  $p$ -groups,

[1] Chapter 4(section 4.3), [5] Chapter 5(section 4)

**Week 10, 11 and 12:** The Sylow theorems and consequences, Applications of Sylow theorems; Classification of groups of order  $p^2$ , where  $p$  is a prime, Classification of groups of  $pq$  where  $p, q$  are distinct primes

[1] Chapter 4(section 4.5) [ 2] Chapter 24, Chapter 9(Theorem 9.7), [5] Chapter 8(section 4 and 5)

**Week 13 and 14:** Finite simple groups, Non simplicity tests; Generalized Cayley's theorem, Index theorem, Embedding theorem and applications. Simplicity of  $A_5$ ,

[2] Chapter 25,

## MME – 501 B : Mathematical Modeling

**Course Objectives:** The main objective of this course is to teach students how to model physical problems using differential equations and solve them. Also, the knowledge of simulation model, linear programming models, graph theoretic models is imparted by which the listed problems can be solved both numerically and analytically.

**Course Learning Outcomes:** The course will enable the students to learn the following:

- 1) Know about power series solution of a differential equation and learn about Legendre's and Bessel's equations.
- 2) Learn about various models such as Monte Carlo simulation models, queuing models and linear programming models.
- 3) Understand the basics of graph theory and learn about social networks, Eulerian and Hamiltonian graphs, diagram tracing puzzles and knight's tour problem.

### Unit 1: Power Series Solutions

(30 marks, 4 weeks)

Power series solution of a differential equation about an ordinary point, Solution about a regular singular point, the method of Frobenius. Legendre's and Bessel's equation.

### Unit 2: Monte Carlo Simulation

(35 marks, 5 weeks)

Monte Carlo Simulation Modeling: Simulating deterministic behavior (area under a curve, volume under a surface); Generating Random Numbers: Middle square method, Linear congruence; Queuing Models: Harbor system, Morning rush hour. Overview of optimization modeling; Linear Programming Model: Geometric solution, Algebraic solution, Simplex method, Sensitivity analysis.

### Unit 3: Graph Theory

(35 marks, 5 weeks)

Graphs, Diagraphs, Networks and subgraphs, Vertex degree, Paths and cycles, Regular and bipartite graphs, Four cube problem, Social networks, Exploring and traveling, Eulerian and Hamiltonian graphs, Applications to dominoes, Diagram tracing puzzles, Knight's tour problem, Gray codes.

#### References:

1. Aldous, Joan M., & Wilson, Robin J. (2007). *Graphs and Applications: An Introductory Approach*. Springer. Indian Reprint.
2. Edwards, C. Henry, Penney, David E., & Calvis, David T. (2015). *Differential Equation and Boundary Value Problems: Computing and Modeling* (5th ed.). Pearson.
3. Giordano, Frank R., Fox, William P., & Horton, Steven B. (2014). *A First Course in Mathematical Modeling* (5th ed.). Brooks/Cole, Cengage Learning.

#### Teaching Plan (MME – 501(B): Mathematical Modeling)

**Weeks 1, 2 and 3:** Power series solution of a differential equation about an ordinary point, Solution about a regular singular point. Legendre's equation. The method of Frobenius.

[2] Chapter 8 (Sections 8.1 to 8.3).

**Week 4:** Bessel's equation. Bessel's function of first kind.

[2] Chapter 8 [Section 8.5 up to Page 553].

**Weeks 5, 6 and 7:** Monte Carlo simulation modeling: Simulating deterministic behavior (area under a curve, volume under a surface), Generating random numbers: Middle square method, Linear congruence. Queuing models: Harbor system, Morning rush hour.

[3] Chapter 5 (Sections 5.1 to 5.2, and 5.5).

**Weeks 8 and 9:** Overview of optimization modeling, Linear programming model: Geometric solution, Algebraic solution, Simplex method, Sensitivity analysis.

[3] Chapter 7.

**Weeks 10, 11 and 12:** Graphs, Diagraphs, Networks and subgraphs, Vertex degree, Paths and cycles, Regular and bipartite graphs, Four cube problem, Social networks.

[1] Chapter 1 (Section 1.1), and Chapter 2.

**Weeks 13 and 14:** Exploring and traveling, Eulerian and Hamiltonian graphs, Applications to dominoes, Diagram tracing puzzles, Knight's tour problem, Gray codes.

[1] Chapter 3.

### MME-501 C : Integral Transforms

**Course Objective:** The main objectives of this course are to teach students to form and solve Fourier series, Laplace transforms and Fourier transforms and use them in solving some physical problems.

**Course Learning Outcomes:** This course will enable the students to:

- i) Learn Fourier series, Euler's formulae, Bessel's inequality, Fourier series in complex form.
- ii) Know about piecewise continuous functions, Dirac's delta function, Laplace transforms and its properties.
- iii) Solve ordinary differential equations using Laplace transforms.
- iv) Familiarise with Fourier transforms of functions belonging to class A, relation between Laplace and Fourier transforms.
- v) Explain Parseval's identity, Plancherel's theorem and applications of Fourier transforms to boundary value problems.



**Unit 1: Fourier Series**

(25 marks, 3 weeks)

Fourier series, Euler's formulae, Dirichlet's conditions for Fourier series, Fourier sine and cosine series, Convergence of Fourier series, Fourier series for even and odd functions, Half-Range Fourier series, Half-Range expansions, Bessel's inequality, The complex form of Fourier series.

**Unit 2: Laplace Transforms**

(40 marks, 6 weeks)

Integral transform, Kernel of an integral transform, Laplace transform, Existence theorem, Linearity property, Shifting theorems, Change of scale property, Laplace transforms of derivatives and integrals, Laplace transforms of periodic functions, Integral equations, Dirac's delta function.

Further properties of Laplace transforms: Multiplication by positive integral power of  $t$ , Division by  $t$ , Inverse Laplace transform, Lerch's theorem, Linearity property of inverse Laplace transform, Inverse transform of derivatives, Convolution theorem, Integral equations, Applications of Laplace transform in obtaining solutions of ordinary differential equations and integral equations.

**Unit 3: Fourier Transforms**

(35 marks, 5 weeks)

Fourier and inverse Fourier transforms, Fourier sine and cosine transforms, Inverse Fourier sine and cosine transforms, Relation between Fourier and Laplace transforms, Linearity property, Change of scale property, Shifting property, Modulation theorem, Solution of integral equation by Fourier sine and cosine transforms, Convolution theorem for Fourier transform, Parseval's identity for Fourier transform, Plancherel's theorem, Fourier transform of derivatives, Applications of infinite Fourier transforms to boundary value problems, Finite Fourier transform, Inversion formula for finite Fourier transforms.

**References:**

1. James Ward Brown & Ruel V. Churchill (2011), *Fourier Series and Boundary Value Problems*. McGraw-Hill Education.
2. Dr. S. Sreenadh, S. Ranganathan, Dr. M. V. S. S. N. Prasad & Dr. V. Ramesh Babu, *Fourier series & Integral transforms* (Reprint 2020), S Chand.
3. MD Raisinghania (2022), *Advanced Differential Equations* (20<sup>th</sup> edition), S Chand and Company Ltd., New Delhi.

**Additional Reading:**

1. Erwin Kreyszig (2011). *Advanced Engineering Mathematics* (10<sup>th</sup> edition). Wiley
2. A. Zygmund (2002). *Trigonometric Series* (3<sup>rd</sup> edition). Cambridge University Press.
3. Phil Dyke (2014). *An Introduction to Laplace Transform and Fourier*. Springer.
4. Rajendra Bhatia. *Fourier Series* (2<sup>nd</sup> edition). Hindustan Book Agency (India), New Delhi.
5. H K Dass (2018). *Advanced Engineering Mathematics* (22<sup>nd</sup> edition). S Chand and Company Ltd., New Delhi.

**Teaching Plan (Theory of MME-501(C): Integral Transforms):**

**Week 1:** Fourier series, Periodic function, Euler's formulae, Dirichlet's Conditions for Fourier series, Convergence of Fourier series. [3] [Part-IIIA, Ch-1(Section1.1- 1.4)]

**Week 2:** Fourier series for even and odd functions, Half- Range Fourier sine and cosine series. Half-Range expansions. [2] [Ch-1 (Section 1.6 to 1.11)]

**Week 3:** Bessel's inequality & the complex form of Fourier series. [2] [Ch-4 Section4.1,[1] Ch-2 Section 9-12]

**Week 4:** Integral transform, Kernel of an integral transform, Laplace transform, Existence theorem, Linearity property.[3] [Part-IVA, Ch-1 (Section 1.1-1.8)]

**Week 5:** Shifting theorems, Change of scale property, Laplace transforms of derivatives and integrals, Further properties of Laplace transforms: Multiplication by positive integral power of t, Division by t. [3] [Part-IVA, Ch-1 (Section 1.11,1.13 to 1.18)]

**Week 6:** Laplace transform of periodic functions, Integral equations & Dirac's delta function. [3] [Part-IVA, Ch-1 (Section 1.20 to 1.22 (II))]

**Week 7:** Inverse Laplace transform, Lerch's theorem, Linearity property of inverse Laplace transform.[3] Part-IVA, [ Ch-2 (Section 2.2, 2.3A & 2.5)]

**Week 8:** Inverse transform of derivatives and integrals, Convolution theorem.[3] [Part-IVA, Ch-2 (Section 2.11, 2.12, & 2.16)]

**Week 9:** Applications of Laplace transform in obtaining solutions of ordinary differential equations and integral equations.[3] [Part-IVA, Ch-3 & 4]

**Week 10:** Fourier and inverse Fourier transforms, Fourier sine and cosine transforms, Inverse Fourier sine and cosine transforms, Relation between Fourier and Laplace transforms.[3] [Part-IVB, Ch-1 (Section 1.6 to 1.9)]

**Week 11:** Linearity property, Change of scale property, Shifting property, Modulation theorem, Solution of integral equation by Fourier sine and cosine transforms.[3] [Part-IVB, Ch-1 (Section 1.11 to 1.14 & 1.17)]

**Week 12:** Convolution theorem for Fourier transform, Parseval's identity for Fourier transform, Plancherel's theorem, Fourier transform of derivatives. [3] [Part-IVB, Ch-1 (Section 1.20 to 1.22)]

**Week 13:** Applications of infinite Fourier transforms to boundary value problems. [3] [Part-IVB, Ch-1 Sec III,]

**Week 14:** Finite Fourier transform, Inversion formula for finite Fourier transforms.[3] [Part-IVB, Ch-2 (Section I & II)]

### **.MME : 602 [SEMESTER – VI]**

#### **ANY ONE OF THE FOLLOWING**

#### **Special Theory of Relativity & Tensors/ Linear Programming and its Applications/ Probability Theory and Statistics**

#### **MME 602 A : Special Theory of Relativity & Tensors**

**Total Marks: 100** (Theory: 75, Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorial (per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** The course aims at providing the basic knowledge of Newtonian mechanics, Relativistic kinematics, Relativistic mechanics and Tensor calculus.

**Course Learning Outcomes:** The course will enable the students to:

- i) Understand the basic elements of Newtonian mechanics including Michelson-Morley experiment and geometrical interpretations of Lorentz transformation equations.
- ii) Learn about length contraction, time dilation and Lorentz contraction factor.
- iii) Study 4-dimensional Minkowskian space-time and its consequences.
- iv) Learn about transformation of co-ordinates, contravariant and covariant tensors.
- v) Understand the algebraic operations of tensors, symmetric and skew-symmetric tensors

**Unit 1: Newtonian Mechanics and Relativistic Kinematics (35 marks, 5 weeks)**

Inertial frames, Galilean transformation, Michelson-Morley experiment, Lorentz-Fitzgerald contraction hypothesis, Relativistic concept of space and time, Postulates of special theory of relativity.

Lorentz transformation equations and its geometrical equations, Group properties and its geometrical equations, consequences of Lorentz transformation equations like Relativity of Simultaneity, Einstein's time dilation, length contraction and related problems, transformation equations for components of velocity and acceleration of a particle.

**Unit 2: Relativistic Mechanics (35 marks, 5 weeks)**

Variation of mass with velocity, Equivalence of mass and energy and its consequences, Transformation equations for mass, momentum, energy and force. Relation between momentum and energy. Energy momentum four vector.

Four dimensional Minkowskian spacetime of special relativity, time-like, light-like and space like intervals, Null cone, proper time, world line of a particle, Four tensors in Minkowskian space-time.

**Unit 3: Tensors (30 Marks, 4 weeks)**

Space of N-dimension, Transformation of co-ordinates, contravariant and covariant vectors (Tensor of first order), Tensor of second order ( or of rank two), Tensors of higher rank (or higher orders), Mixed tensors, Kronecker delta symbol, Invariant or scalar, Algebraic operations with tensors, Addition & subtraction of tensors, contraction, product of tensors, Inner Product, symmetric and Skew symmetric tensor.

**References:**

1. **Farook Rahaman (2014):** The Special Theory of Relativity, A Mathematical Approach, Springer
2. **Robert Resnick (2007):** Introduction to Special Relativity, John Wiley
3. **James L Anderson (1973):** Introduction to the Theory of Relativity, Dover Publications

**Additional Reading:**

1. **M. Ray:** Special Theory of Relativity, S Chand and Co
2. **A. Das (1993):** The Special Theory of Relativity, Springer
3. **Banerjee and Banerjee (2012):** The Special Theory of Relativity, PHI, New Delhi.
4. **Dirac :** General Theory of Relativity, Prentice Hall of India, New Delhi.
5. **S.K. Bose:** General Theory of Relativity, Wiley Eastern Ltd.

**Teaching plan {Theory of MME-602(A)}:** Special Theory of Relativity and tensors.

**Week 1:** Inertial frames, Galilean transformation[1] chapter 1 (section 1.1, 1.2,1.3) Michelson-Morley experiment [1] chapter 2 (section 2.2) Lorentz-Fitzgerald contraction hypothesis [1] chapter2 (section2.4)

**Week 2:** Relativistic concept of space &time[1] chapter 2 (section2.5)  
Postulates of Special Theory of Relativity [1] chapter 3 (section3.1)

**Week 3:** Lorentz transformation equations and its geometrical equations[1] chapter 3 {section(3.2) 3.2.1}  
Group properties of Lorentz transformation.

**Week 4:** Consequences of Lorentz equations like Relativity of Simulaniety[1] chapter 4 (section4.3), Einstein's time dilation[1] chapter 4 (section4.2) length contraction[1] chapter 4 (section 4.1)

**Week 5:** Transformation equations for components of velocity [1] chapter 7 (section7.1, section 7.2), acceleration of a particle[1] chapter 7 (section 7.3, section 7.4)

**Week 6:** Variation of mass with velocity [2] chapter 3 (section3.3) Equivalence of mass and energy and its consequences[1] chapter 10 (section10.4)

**Week 7:** Transformation equations for mass, momentum, energy and force [2] chapter 3 (section3.7)

**Week 8:** Relation between momentum and energy [1] chapter 10 (section 10.5, section 11.11) Energy momentum four vector [1] chapter 10 (10.2)

**Week 9:** Four dimensional Minkowskian space-time of special relativity, time-like, light-like and space like intervals, Null cone [1] chapter 8 (section8.1)

**Week 10:** Proper time [1] chapter 8 (section8.2) World line of a particle, Four vectors in Minkowskian space-time [1] chapter 8 (section8.4)

## **MME – 602 B : Linear Programming and its Applications**

**Total Marks:** 100 (Theory: 75 and Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorial (per week)

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** This course develops the ideas underlying the Simplex Method for Linear Programming Problem, as an important branch of Operations Research. The course covers Linear Programming with applications to Transportation, Assignment and Game Problem. Such problems arise in manufacturing resource planning and financial sectors.

**Course Learning Outcomes:** This course will enable the students to learn:

- i) Analyze and solve linear programming models of real life situations.
- ii) The graphical solution of LPP with only two variables, and illustrate the concept of convex set and extreme points. The theory of the simplex method is developed.
- iii) The relationships between the primal and dual problems and their solutions with applications to transportation, assignment and two-person zero-sum game problem.

### **Unit 1: Introduction to Linear Programming**

**(30 marks, 4 weeks)**

The Linear Programming Problem: Standard, Canonical and matrix forms, Graphical solution. Convex and polyhedral sets, Hyperplanes, Extreme points. Basic solutions; Basic Feasible Solutions; Reduction of any feasible solution to a basic feasible solution; Correspondence between basic feasible solutions and extreme points.

### **Unit 2: Methods of Solving Linear Programming Problem and dual problem (40 marks, 6 weeks)**

Simplex Method: Algebra of Simplex method, Optimal solution, Termination criteria for optimal solution of the Linear Programming Problem, Unique and alternate optimal solutions, Unboundedness; Simplex Algorithm and its Tableau Format; Artificial variables, Two-phase method, Big-M method.

Motivation and Formulation of Dual problem; Primal-Dual relationships; Fundamental Theorem of Duality; Complimentary Slackness.

### **Unit 3: Applications**

**(30 marks, 4 weeks)**

*Transportation Problem:* Definition and formulation; Methods of finding initial basic feasible solutions; North West corner rule. Least cost method; Vogel's Approximation method; Algorithm for solving Transportation Problem.

*Assignment Problem:* Mathematical formulation and Hungarian method of solving.

*Game Theory:* Basic concept, Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear Programming method of solving a game.

**References:**

1. Bazaraa, Mokhtar S., Jarvis, John J., & Sherali, Hanif D. (2010). *Linear Programming and Network Flows* (4th ed.). John Wiley and Sons.
2. Hadley, G. (1997). *Linear Programming*. Narosa Publishing House. New Delhi.
3. Taha, Hamdy A. (2010). *Operations Research: An Introduction* (9th ed.). Pearson.

**Additional Readings:**

1. Hillier, F. S. & Lieberman, G. J. (2010). *Introduction to Operations Research- Concepts and Cases* (9th ed.). Tata McGraw Hill.
2. Thie, Paul R., & Keough, G. E. (2014). *An Introduction to Linear Programming and Game Theory*. (3rd ed.). Wiley India Pvt. Ltd.

**Teaching Plan ( MME-602(B)): Linear Programming and its Applications**

**Week 1:** The Linear Programming Problem: Standard, Canonical and matrix forms, Graphical solution.

[1] Chapter 1(section 1.1) [2] Chapter 1 (sections 1.1 to 1.4 and 1.6)

**Week 2&3:** Convex and polyhedral sets, Hyperplanes, Extreme points. Basic solutions; Basic Feasible Solutions.

[1] Chapter 2 (section 2.4 to 2.7) Chapter 3(section 3.2), [2] Chapter 2(section 2.16, 2.19, 2.20)

**Week 4:** Reduction of any feasible solution to a basic feasible solution; Correspondence between basic feasible solutions and extreme points.

[1] Chapter 3(section 3.2), [2] Chapter 3(section 3.4, 3.5,3.6, 3.10)

**Week 5:** Simplex Method: Algebra of Simplex method, Optimal solution, Termination criteria for optimal solution of the Linear Programming Problem, Unique and alternate optimal solutions, Unboundedness.

[1] Chapter 3(section 3.3, 3.5 3.6), [2] Chapter 3(section 3.7, 3.8, 3.9)

**Week 6 :** Simplex Algorithm and its Tableau Format.

[1] Chapter 3(section 3.7, 3.8) [2] Chapter 4(section 4.8, to 4.11)

**Week 7:** Artificial variables, Two-phase method, Big-M method.

[1] Chapter 4(section 4.1 to 4.3) [2] Chapter 5(section 5.1 to 5.5)

**Week 8 to 10:** Motivation and Formulation of Dual problem; Primal-Dual relationships; Fundamental Theorem of Duality; Complimentary Slackness theorem with examples.

[1] Chapter 6(section 6.1, 6.2) [2] Chapter 8(section 8.1 to 8.5)

**Week 11 to 12:** Transportation problem, Assignment Problem

[3] Chapter 5(section 5.1, 5.3, 5.4)

**Week 13 to 14:** *Game Theory*: Basic concept, Formulation and solution of two-person zero-sum games, Games with mixed strategies, Linear Programming method of solving a game.

[3] Chapter 11(section 11.12, 11.13)

## **MME-602 C : Probability Theory and Statistics**

**Total Marks:** 100 (Theory: 75 + Internal Assessment: 25)

**Workload:** 5 Lectures (per week), 1 Tutorial (per week )

**Duration:** 14 Weeks (70 Hrs.) **Examination:** 3 Hrs.

**Course Objectives:** To make the students familiar with the basic statistical concepts and tools which are needed to study situations involving uncertainty or randomness. The course intends to render the students to several examples and exercises that blend their everyday experiences with their scientific interests.

**Course Learning Outcomes:** This course will enable the students to learn:

- i) Distributions to study the joint behavior of two random variables.
- ii) To establish a formulation helping to predict one variable in terms of the other, i.e., correlation and linear regression.
- iii) Central limit theorem, which helps to understand the remarkable fact that: the empirical frequencies of so many natural populations, exhibit a bell shaped curve.

**Unit 1: Probability Functions and Moment Generating Function (30 marks, 4 weeks)**

Sample space, Probability set function, Real random variables – Discrete and continuous, Cumulative distribution function, Probability mass/density functions, Transformations, Mathematical expectation, Moments, Moment generating function, Characteristic function.

**Unit 2: Univariate Discrete, Continuous Distributions and Bivariate distributions (35 marks, 5 weeks)**

**Discrete distributions:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson;

**Continuous distributions:** Uniform, Gamma, Exponential, Chi-square, Beta and Normal; Normal approximation to the binomial distribution.

**Bivariate Distribution:** Joint cumulative distribution function and its properties, Joint probability density function, Marginal distributions, Expectation of function of two random variables, Joint moment generating function, Conditional distributions and expectations.

**Unit 3: Correlation, Regression and Central Limit Theorem (35 marks, 5 weeks)**

The Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function, Independent random variables, Linear regression for two variables, The method of least squares, Bivariate normal distribution, Chebyshev's theorem, Strong law of large numbers, Central limit theorem and weak law of large numbers.

**References:**

1. Hogg, Robert V., McKean, Joseph W., & Craig, Allen T. (2013). *Introduction to Mathematical Statistics*(7<sup>th</sup> ed.). Pearson Education, Inc.
2. Miller, Irwin & Miller, Marylees. (2014). John E. Freund's *Mathematical Statistics with Applications* (8<sup>th</sup> ed.). Pearson. Dorling Kindersley (India).

**Additional Reading:**

1. Ross, Sheldon M. (2014). *Introduction to Probability Models* (11<sup>th</sup> ed.). Elsevier Inc.AP.
2. Mood, A.M., Graybill, F.A. & Boes, D.C. (1974). *Introduction to the Theory of Statistics* (3<sup>rd</sup> ed.). McGraw-Hill Education Pvt. Ltd. Indian edition (2017).

**Teaching Plan (MME-602 (C): Probability Theory and Statistics**

**Week 1:** Sample space, Probability set function, Real random variables – Discrete and continuous. [1] [Chapter-1(Section 1.1, 1.3, 1.5 to 1.7).

**Week 2:** Cumulative distribution function, Probability mass/density functions. [2] Chapter -2 Section 2 & 4).

**Week 3:** Transformations, Mathematical expectation. [1] Ch-1 (Section 1.7.2&1.8).

**Week 4:** Moments, Moment generating function, Characteristic function. [1] Chapter-1 (Section 1.9).

**Week 5: Discrete distributions:** Uniform, Bernoulli, Binomial, Negative binomial, Geometric and Poisson. [2] Chapter-5 (Section 2 to 5 & 7).

**Week 6: Continuous distributions:** Uniform, Gamma, Exponential, Chi-square, Beta and Normal; Normal approximation to the binomial distribution. [2] Chapter-6 (Section 2 to 6).

**Week 7: Bivariate Distribution:** Joint cumulative distribution function and its properties, Joint probability density function. [1] Chapter-2 (Section 2.1).

**Week 8:** Marginal distributions, Expectation of function of two random variables, Joint moment generating function.

[1] Chapter-2 (Section 2.1.1, 2.1.2).

**Week 9:** Conditional distributions and expectations.

[1] Chapter-2 (Section 2.3).

**Week 10:** The Correlation coefficient, Covariance, Calculation of covariance from joint moment generating function.

[1] Chapter -2 (Section 2.5, 2.5.1).

**Week 11:** Independent random variables, Linear regression for two variables.

[1] Chapter-2 (Section 2.4) & [2] Ch-6 (Section 2).

**Week 12:** The method of least squares, Bivariate normal distribution.

[2] Chapter-6 (Section 3) & [2] Ch-5 (Section 7).

**Week 13:** Chebyshev's theorem, Strong law of large numbers.

[2] Chapter-3 (Section 4).

**Week 14:** Central limit theorem and weak law of large numbers.

[1] Chapter-5 (Section 5.3, 5.4).